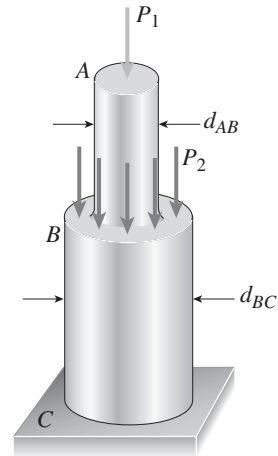


Tension, Compression, and Shear

Normal Stress and Strain

Problem 1.2-1 A solid circular post ABC (see figure) supports a load $P_1 = 2500$ lb acting at the top. A second load P_2 is uniformly distributed around the shelf at B . The diameters of the upper and lower parts of the post are $d_{AB} = 1.25$ in. and $d_{BC} = 2.25$ in., respectively.

- Calculate the normal stress σ_{AB} in the upper part of the post.
- If it is desired that the lower part of the post have the same compressive stress as the upper part, what should be the magnitude of the load P_2 ?



Solution 1.2-1 Circular post in compression

$$P_1 = 2500 \text{ lb}$$

$$d_{AB} = 1.25 \text{ in.}$$

$$d_{BC} = 2.25 \text{ in.}$$

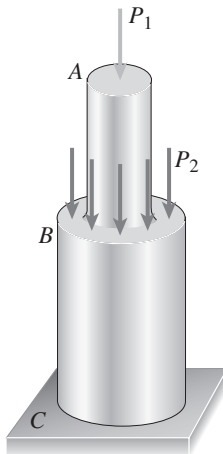
(a) NORMAL STRESS IN PART AB

$$\sigma_{AB} = \frac{P_1}{A_{AB}} = \frac{2500 \text{ lb}}{\frac{\pi}{4}(1.25 \text{ in.})^2} = 2040 \text{ psi} \leftarrow$$

(b) LOAD P_2 FOR EQUAL STRESSES

$$\begin{aligned} \sigma_{BC} &= \frac{P_1 + P_2}{A_{BC}} = \frac{2500 \text{ lb} + P_2}{\frac{\pi}{4}(2.25 \text{ in.})^2} \\ &= \sigma_{AB} = 2040 \text{ psi} \end{aligned}$$

$$\text{Solve for } P_2: P_2 = 5600 \text{ lb} \leftarrow$$



ALTERNATE SOLUTION

$$\sigma_{BC} = \frac{P_1 + P_2}{A_{BC}} = \frac{P_1 + P_2}{\frac{\pi}{4}d_{BC}^2}$$

$$\sigma_{AB} = \frac{P_1}{A_{AB}} = \frac{P_1}{\frac{\pi}{4}d_{AB}^2} \quad \sigma_{BC} = \sigma_{AB}$$

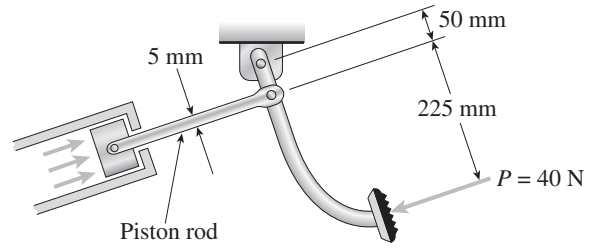
$$\frac{P_1 + P_2}{d_{BC}^2} = \frac{P_1}{d_{AB}^2} \quad \text{or} \quad P_2 = P_1 \left[\left(\frac{d_{BC}}{d_{AB}} \right)^2 - 1 \right]$$

$$\frac{d_{BC}}{d_{AB}} = 1.8$$

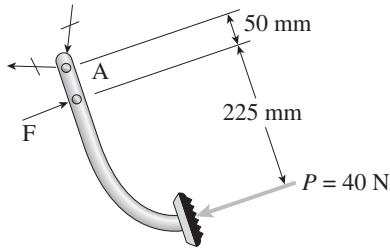
$$\therefore P_2 = 2.24 P_1 = 5600 \text{ lb} \leftarrow$$

Problem 1.2-2 Calculate the compressive stress σ_c in the circular piston rod (see figure) when a force $P = 40 \text{ N}$ is applied to the brake pedal.

Assume that the line of action of the force P is parallel to the piston rod, which has diameter 5 mm. Also, the other dimensions shown in the figure (50 mm and 225 mm) are measured perpendicular to the line of action of the force P .



Solution 1.2-2 Free-body diagram of brake pedal



F = compressive force in piston rod
 d = diameter of piston rod
 = 5 mm

EQUILIBRIUM OF BRAKE PEDAL

$$\Sigma M_A = 0 \quad \curvearrowright \quad \curvearrowleft$$

$$F(50 \text{ mm}) - P(275 \text{ mm}) = 0$$

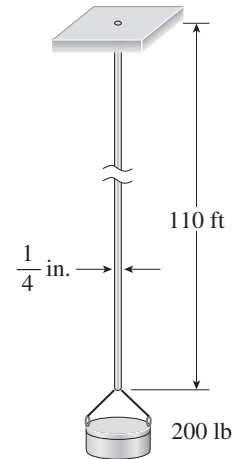
$$F = P \left(\frac{275 \text{ mm}}{50 \text{ mm}} \right) = (40 \text{ N}) \left(\frac{275}{50} \right) = 220 \text{ N}$$

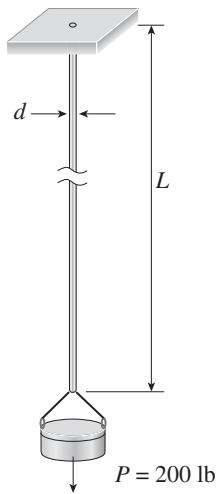
COMPRESSIVE STRESS IN PISTON ROD ($d = 5 \text{ mm}$)

$$\sigma_c = \frac{F}{A} = \frac{220 \text{ N}}{\frac{\pi}{4}(5 \text{ mm})^2} = 11.2 \text{ MPa} \quad \longleftarrow$$

Problem 1.2-3 A steel rod 110 ft long hangs inside a tall tower and holds a 200-pound weight at its lower end (see figure).

If the diameter of the circular rod is $\frac{1}{4}$ inch, calculate the maximum normal stress σ_{\max} in the rod, taking into account the weight of the rod itself. (Obtain the weight density of steel from Table H-1, Appendix H.)



Solution 1.2-3 Long steel rod in tension

$$P = 200 \text{ lb}$$

$$L = 110 \text{ ft}$$

$$d = \frac{1}{4} \text{ in.}$$

$$\text{Weight density: } \gamma = 490 \text{ lb/ft}^3$$

$$W = \text{Weight of rod}$$

$$= \gamma(\text{Volume})$$

$$= \gamma AL$$

$$P = 200 \text{ lb}$$

$$\sigma_{\max} = \frac{W + P}{A} = \gamma L + \frac{P}{A}$$

$$\begin{aligned} \gamma L &= (490 \text{ lb/ft}^3)(110 \text{ ft}) \left(\frac{1}{144} \frac{\text{ft}^2}{\text{in.}^2} \right) \\ &= 374.3 \text{ psi} \end{aligned}$$

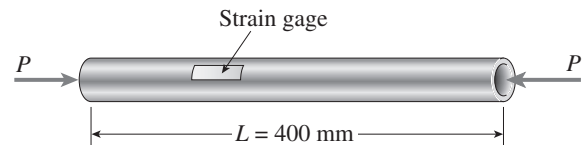
$$\frac{P}{A} = \frac{200 \text{ lb}}{\frac{\pi}{4}(0.25 \text{ in.})^2} = 4074 \text{ psi}$$

$$\sigma_{\max} = 374 \text{ psi} + 4074 \text{ psi} = 4448 \text{ psi}$$

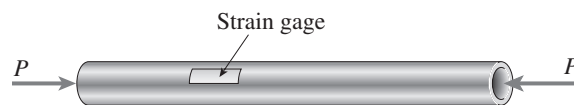
Rounding, we get

$$\sigma_{\max} = 4450 \text{ psi} \quad \leftarrow$$

Problem 1.2-4 A circular aluminum tube of length $L = 400$ mm is loaded in compression by forces P (see figure). The outside and inside diameters are 60 mm and 50 mm, respectively. A strain gage is placed on the outside of the bar to measure normal strains in the longitudinal direction.



- If the measured strain is $\epsilon = 550 \times 10^{-6}$, what is the shortening δ of the bar?
- If the compressive stress in the bar is intended to be 40 MPa, what should be the load P ?

Solution 1.2-4 Aluminum tube in compression

$$\epsilon = 550 \times 10^{-6}$$

$$L = 400 \text{ mm}$$

$$d_2 = 60 \text{ mm}$$

$$d_1 = 50 \text{ mm}$$

(a) SHORTENING δ OF THE BAR

$$\delta = \epsilon L = (550 \times 10^{-6})(400 \text{ mm})$$

$$= 0.220 \text{ mm} \quad \leftarrow$$

(b) COMPRESSIVE LOAD P

$$\sigma = 40 \text{ MPa}$$

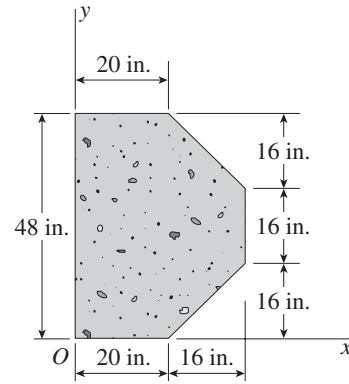
$$\begin{aligned} A &= \frac{\pi}{4}[d_2^2 - d_1^2] = \frac{\pi}{4}[(60 \text{ mm})^2 - (50 \text{ mm})^2] \\ &= 863.9 \text{ mm}^2 \end{aligned}$$

$$P = \sigma A = (40 \text{ MPa})(863.9 \text{ mm}^2)$$

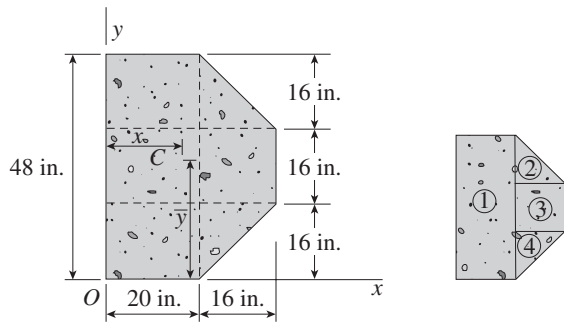
$$= 34.6 \text{ kN} \quad \leftarrow$$

Problem 1.2-5 The cross section of a concrete pier that is loaded uniformly in compression is shown in the figure.

- (a) Determine the average compressive stress σ_c in the concrete if the load is equal to 2500 k.
- (b) Determine the coordinates \bar{x} and \bar{y} of the point where the resultant load must act in order to produce uniform normal stress.



Solution 1.2-5 Concrete pier in compression



USE THE FOLLOWING AREAS:

$$A_1 = (48 \text{ in.})(20 \text{ in.}) = 960 \text{ in.}^2$$

$$A_2 = A_4 = \frac{1}{2}(16 \text{ in.})(16 \text{ in.}) = 128 \text{ in.}^2$$

$$A_3 = (16 \text{ in.})(16 \text{ in.}) = 256 \text{ in.}^2$$

$$\begin{aligned} A &= A_1 + A_2 + A_3 + A_4 \\ &= (960 + 128 + 256 + 128) \text{ in.}^2 \\ &= 1472 \text{ in.}^2 \end{aligned}$$

(a) AVERAGE COMPRESSIVE STRESS σ_c

$$P = 2500 \text{ k}$$

$$\sigma_c = \frac{P}{A} = \frac{2500 \text{ k}}{1472 \text{ in.}^2} = 1.70 \text{ ksi} \leftarrow$$

(b) COORDINATES OF CENTROID c

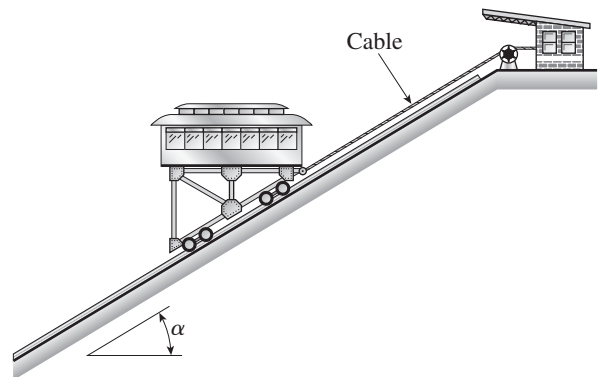
$$\text{From symmetry, } \bar{y} = \frac{1}{2}(48 \text{ in.}) = 24 \text{ in.} \leftarrow$$

$$\bar{x} = \frac{\sum \bar{x}_i A_i}{A} \text{ (see Chapter 12, Eq. 12 - 7a)}$$

$$\begin{aligned} \bar{x} &= \frac{1}{A} (\bar{x}_1 A_1 + 2\bar{x}_2 A_2 + \bar{x}_3 A_3) \\ &= \frac{1}{1472 \text{ in.}^2} [(10 \text{ in.})(960 \text{ in.}^2) \\ &\quad + 2(25.333 \text{ in.})(128 \text{ in.}^2) \\ &\quad + (28 \text{ in.})(256 \text{ in.}^2)] \\ &= 15.8 \text{ in.} \leftarrow \end{aligned}$$

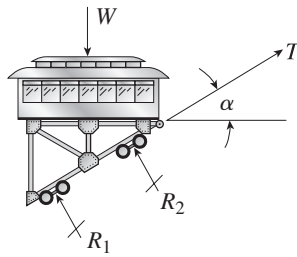
Problem 1.2-6 A car weighing 130 kN when fully loaded is pulled slowly up a steep inclined track by a steel cable (see figure). The cable has an effective cross-sectional area of 490 mm², and the angle α of the incline is 30°.

Calculate the tensile stress σ_t in the cable.



Solution 1.2-6 Car on inclined track

FREE-BODY DIAGRAM OF CAR

 $W =$ Weight of car $T =$ Tensile force in cable $\alpha =$ Angle of incline $A =$ Effective area of cable $R_1, R_2 =$ Wheel reactions (no friction force between wheels and rails)

EQUILIBRIUM IN THE INCLINED DIRECTION

$$\Sigma F_T = 0 \quad \nearrow \swarrow \quad T - W \sin \alpha = 0$$

$$T = W \sin \alpha$$

TENSILE STRESS IN THE CABLE

$$\sigma_t = \frac{T}{A} = \frac{W \sin \alpha}{A}$$

SUBSTITUTE NUMERICAL VALUES:

$$W = 130 \text{ kN} \quad \alpha = 30^\circ$$

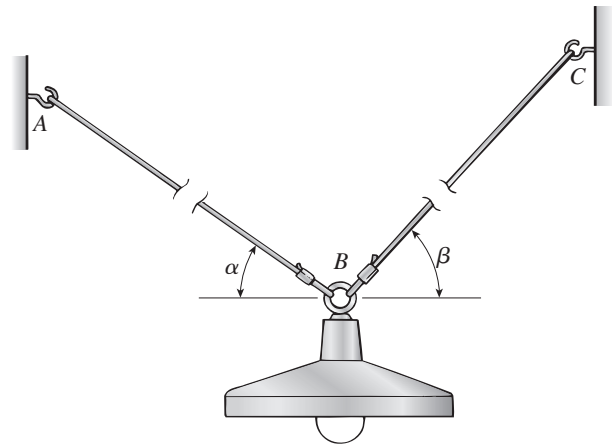
$$A = 490 \text{ mm}^2$$

$$\sigma_t = \frac{(130 \text{ kN})(\sin 30^\circ)}{490 \text{ mm}^2}$$

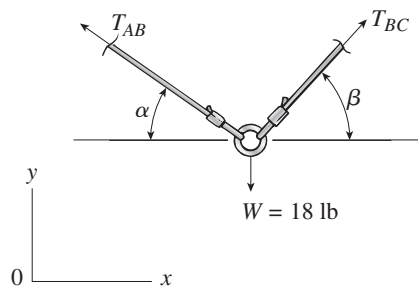
$$= 133 \text{ MPa} \quad \longleftarrow$$

Problem 1.2-7 Two steel wires, AB and BC , support a lamp weighing 18 lb (see figure). Wire AB is at an angle $\alpha = 34^\circ$ to the horizontal and wire BC is at an angle $\beta = 48^\circ$. Both wires have diameter 30 mils. (Wire diameters are often expressed in mils; one mil equals 0.001 in.)

Determine the tensile stresses σ_{AB} and σ_{BC} in the two wires.

**Solution 1.2-7 Two steel wires supporting a lamp**

FREE-BODY DIAGRAM OF POINT B



$$\alpha = 34^\circ \quad \beta = 48^\circ$$

$$d = 30 \text{ mils} = 0.030 \text{ in.}$$

$$A = \frac{\pi d^2}{4} = 706.9 \times 10^{-6} \text{ in.}^2$$

SUBSTITUTE NUMERICAL VALUES:

$$-T_{AB}(0.82904) + T_{BC}(0.66913) = 0$$

$$T_{AB}(0.55919) + T_{BC}(0.74314) - 18 = 0$$

SOLVE THE EQUATIONS:

$$T_{AB} = 12.163 \text{ lb} \quad T_{BC} = 15.069 \text{ lb}$$

TENSILE STRESSES IN THE WIRES

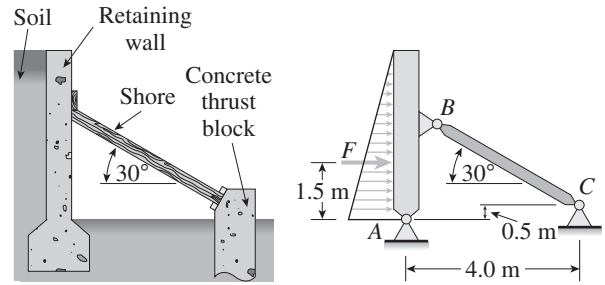
$$\sigma_{AB} = \frac{T_{AB}}{A} = 17,200 \text{ psi} \quad \longleftarrow$$

$$\sigma_{BC} = \frac{T_{BC}}{A} = 21,300 \text{ psi} \quad \longleftarrow$$

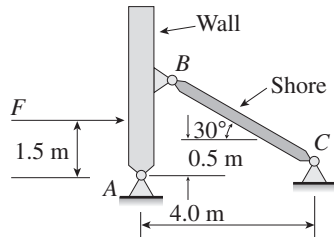
Problem 1.2-8 A long retaining wall is braced by wood shores set at an angle of 30° and supported by concrete thrust blocks, as shown in the first part of the figure. The shores are evenly spaced, 3 m apart.

For analysis purposes, the wall and shores are idealized as shown in the second part of the figure. Note that the base of the wall and both ends of the shores are assumed to be pinned. The pressure of the soil against the wall is assumed to be triangularly distributed, and the resultant force acting on a 3-meter length of the wall is $F = 190$ kN.

If each shore has a $150 \text{ mm} \times 150 \text{ mm}$ square cross section, what is the compressive stress σ_c in the shores?



Solution 1.2-8 Retaining wall braced by wood shores



$$F = 190 \text{ kN}$$

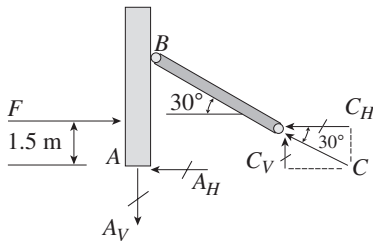
$$A = \text{area of one shore}$$

$$A = (150 \text{ mm})(150 \text{ mm})$$

$$= 22,500 \text{ mm}^2$$

$$= 0.0225 \text{ m}^2$$

FREE-BODY DIAGRAM OF WALL AND SHORE



SUMMATION OF MOMENTS ABOUT POINT A

$$\Sigma M_A = 0 \quad \curvearrowright \quad \curvearrowleft$$

$$-F(1.5 \text{ m}) + C_V(4.0 \text{ m}) + C_H(0.5 \text{ m}) = 0$$

or

$$-(190 \text{ kN})(1.5 \text{ m}) + C(\sin 30^\circ)(4.0 \text{ m}) + C(\cos 30^\circ)(0.5 \text{ m}) = 0$$

$$\therefore C = 117.14 \text{ kN}$$

C = compressive force in wood shore

C_H = horizontal component of C

C_V = vertical component of C

$C_H = C \cos 30^\circ$

$C_V = C \sin 30^\circ$

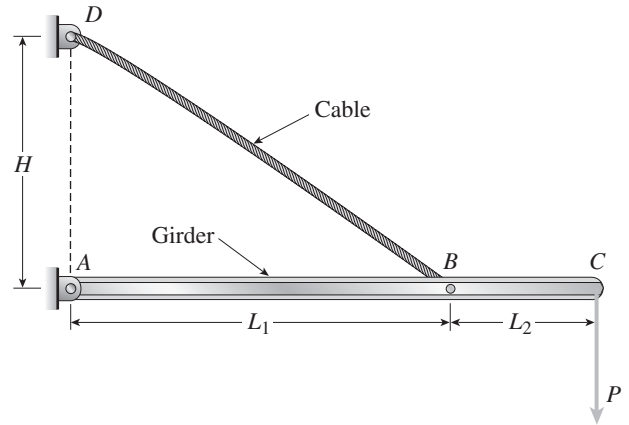
COMPRESSIVE STRESS IN THE SHORES

$$\sigma_c = \frac{C}{A} = \frac{117.14 \text{ kN}}{0.0225 \text{ m}^2}$$

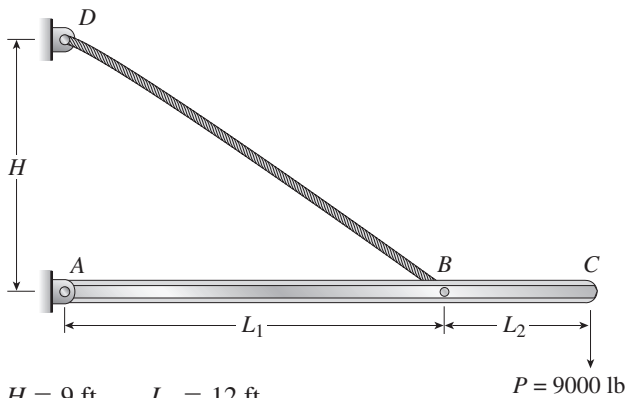
$$= 5.21 \text{ MPa} \quad \longleftarrow$$

Problem 1.2-9 A loading crane consisting of a steel girder ABC supported by a cable BD is subjected to a load P (see figure). The cable has an effective cross-sectional area $A = 0.471 \text{ in}^2$. The dimensions of the crane are $H = 9 \text{ ft}$, $L_1 = 12 \text{ ft}$, and $L_2 = 4 \text{ ft}$.

- If the load $P = 9000 \text{ lb}$, what is the average tensile stress in the cable?
- If the cable stretches by 0.382 in. , what is the average strain?

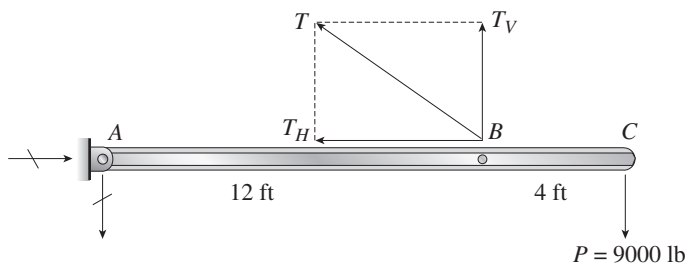


Solution 1.2-9 Loading crane with girder and cable



$H = 9 \text{ ft}$ $L_1 = 12 \text{ ft}$
 $L_2 = 4 \text{ ft}$ $A = \text{effective area of cable}$
 $A = 0.471 \text{ in}^2$
 $P = 9000 \text{ lb}$

FREE-BODY DIAGRAM OF GIRDER



$T = \text{tensile force in cable}$
 $P = 9000 \text{ lb}$

EQUILIBRIUM

$$\sum M_A = 0 \quad \curvearrowright \quad \curvearrowleft$$

$$T_V (12 \text{ ft}) - (9000 \text{ lb})(16 \text{ ft}) = 0$$

$$T_V = 12,000 \text{ lb}$$

$$\frac{T_H}{T_V} = \frac{L_1}{H} = \frac{12 \text{ ft}}{9 \text{ ft}}$$

$$\therefore T_H = T_V \left(\frac{12}{9} \right)$$

$$T_H = (12,000 \text{ lb}) \left(\frac{12}{9} \right)$$

$$= 16,000 \text{ lb}$$

TENSILE FORCE IN CABLE

$$T = \sqrt{T_H^2 + T_V^2} = \sqrt{(16,000 \text{ lb})^2 + (12,000 \text{ lb})^2}$$

$$= 20,000 \text{ lb}$$

(a) AVERAGE TENSILE STRESS IN CABLE

$$\sigma = \frac{T}{A} = \frac{20,000 \text{ lb}}{0.471 \text{ in}^2} = 42,500 \text{ psi} \quad \leftarrow$$

(b) AVERAGE STRAIN IN CABLE

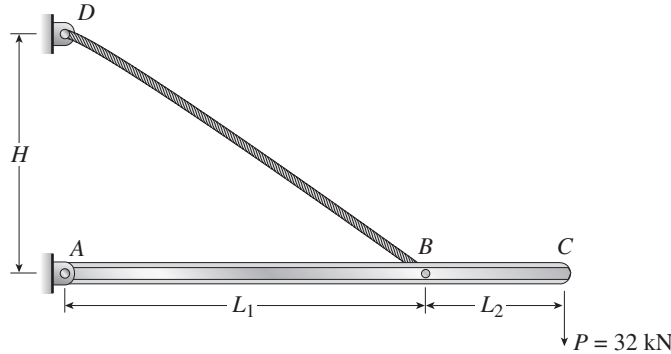
$$L = \text{length of cable} \quad L = \sqrt{H^2 + L_1^2} = 15 \text{ ft}$$

$$\delta = \text{stretch of cable} \quad \delta = 0.382 \text{ in.}$$

$$\varepsilon = \frac{\delta}{L} = \frac{0.382 \text{ in.}}{(15 \text{ ft})(12 \text{ in./ft})} = 2120 \times 10^{-6} \quad \leftarrow$$

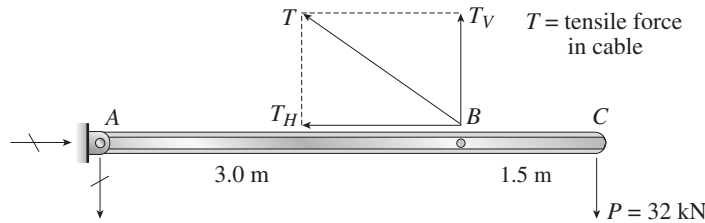
Problem 1.2-10 Solve the preceding problem if the load $P = 32$ kN; the cable has effective cross-sectional area $A = 481$ mm²; the dimensions of the crane are $H = 1.6$ m, $L_1 = 3.0$ m, and $L_2 = 1.5$ m; and the cable stretches by 5.1 mm. Figure is with Prob. 1.2-9.

Solution 1.2-10 Loading crane with girder and cable



$$\begin{aligned}
 H &= 1.6 \text{ m} & L_1 &= 3.0 \text{ m} \\
 L_2 &= 1.5 \text{ m} & A &= \text{effective area of cable} \\
 A &= 481 \text{ mm}^2 & P &= 32 \text{ kN}
 \end{aligned}$$

FREE-BODY DIAGRAM OF GIRDER



EQUILIBRIUM

$$\begin{aligned}
 \Sigma M_A &= 0 \quad (\oplus \curvearrowright) \\
 T_V(3.0 \text{ m}) - (32 \text{ kN})(4.5 \text{ m}) &= 0 \\
 T_V &= 48 \text{ kN} \\
 \frac{T_H}{T_V} &= \frac{L_1}{H} = \frac{3.0 \text{ m}}{1.6 \text{ m}} \\
 \therefore T_H &= T_V \left(\frac{3.0}{1.6} \right) \\
 T_H &= (48 \text{ kN}) \left(\frac{3.0}{1.6} \right) \\
 &= 90 \text{ kN}
 \end{aligned}$$

TENSILE FORCE IN CABLE

$$\begin{aligned}
 T &= \sqrt{T_H^2 + T_V^2} = \sqrt{(90 \text{ kN})^2 + (48 \text{ kN})^2} \\
 &= 102 \text{ kN}
 \end{aligned}$$

(a) AVERAGE TENSILE STRESS IN CABLE

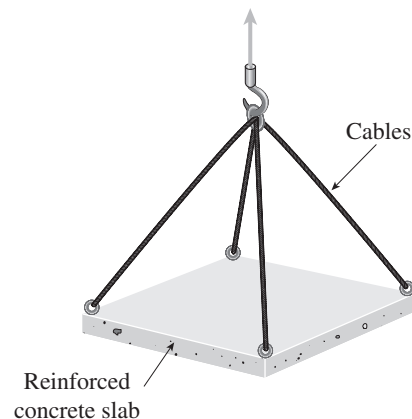
$$\sigma = \frac{T}{A} = \frac{102 \text{ kN}}{481 \text{ mm}^2} = 212 \text{ MPa} \leftarrow$$

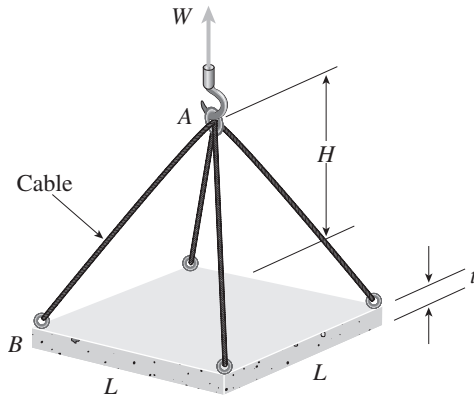
(b) AVERAGE STRAIN IN CABLE

$$\begin{aligned}
 L &= \text{length of cable} \\
 L &= \sqrt{H^2 + L_1^2} = 3.4 \text{ m} \\
 \delta &= \text{stretch of cable} \\
 \delta &= 5.1 \text{ mm} \\
 \epsilon &= \frac{\delta}{L} = \frac{5.1 \text{ mm}}{3.4 \text{ m}} = 1500 \times 10^{-6} \leftarrow
 \end{aligned}$$

Problem 1.2-11 A reinforced concrete slab 8.0 ft square and 9.0 in. thick is lifted by four cables attached to the corners, as shown in the figure. The cables are attached to a hook at a point 5.0 ft above the top of the slab. Each cable has an effective cross-sectional area $A = 0.12$ in².

Determine the tensile stress σ_t in the cables due to the weight of the concrete slab. (See Table H-1, Appendix H, for the weight density of reinforced concrete.)



Solution 1.2-11 Reinforced concrete slab supported by four cables

H = height of hook above slab

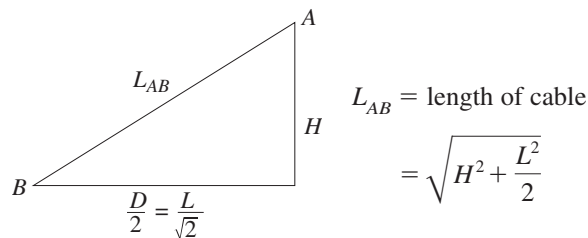
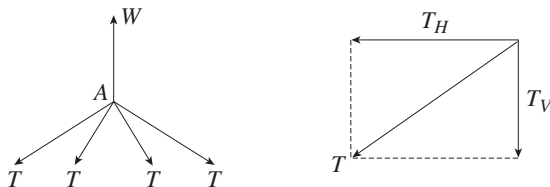
L = length of side of square slab

t = thickness of slab

γ = weight density of reinforced concrete

W = weight of slab = $\gamma L^2 t$

D = length of diagonal of slab = $L\sqrt{2}$

DIMENSIONS OF CABLE AB**FREE-BODY DIAGRAM OF HOOK AT POINT A**

T = tensile force in a cable

Cable AB:

$$\frac{T_V}{T} = \frac{H}{L_{AB}}$$

$$T_V = T \left(\frac{H}{\sqrt{H^2 + L^2/2}} \right) \quad (\text{Eq. 1})$$

EQUILIBRIUM

$$\Sigma F_{\text{vert}} = 0 \uparrow + \downarrow -$$

$$W - 4T_V = 0$$

$$T_V = \frac{W}{4} \quad (\text{Eq. 2})$$

COMBINE EQS. (1) & (2):

$$T \left(\frac{H}{\sqrt{H^2 + L^2/2}} \right) = \frac{W}{4}$$

$$T = \frac{W}{4} \frac{\sqrt{H^2 + L^2/2}}{H} = \frac{W}{4} \sqrt{1 + L^2/2H^2}$$

TENSILE STRESS IN A CABLE

A = effective cross-sectional area of a cable

$$\sigma_t = \frac{T}{A} = \frac{W}{4A} \sqrt{1 + L^2/2H^2} \leftarrow$$

SUBSTITUTE NUMERICAL VALUES AND OBTAIN σ_t :

$$H = 5.0 \text{ ft} \quad L = 8.0 \text{ ft} \quad t = 9.0 \text{ in.} = 0.75 \text{ ft}$$

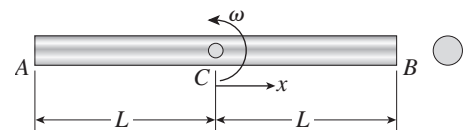
$$\gamma = 150 \text{ lb/ft}^3 \quad A = 0.12 \text{ in.}^2$$

$$W = \gamma L^2 t = 7200 \text{ lb}$$

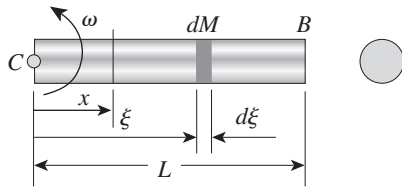
$$\sigma_t = \frac{W}{4A} \sqrt{1 + L^2/2H^2} = 22,600 \text{ psi} \leftarrow$$

Problem 1.2-12 A round bar ACB of length $2L$ (see figure) rotates about an axis through the midpoint C with constant angular speed ω (radians per second). The material of the bar has weight density γ .

- Derive a formula for the tensile stress σ_x in the bar as a function of the distance x from the midpoint C .
- What is the maximum tensile stress σ_{max} ?



Solution 1.2-12 Rotating Bar



ω = angular speed (rad/s)

A = cross-sectional area

γ = weight density

$\frac{\gamma}{g}$ = mass density

We wish to find the axial force F_x in the bar at Section D, distance x from the midpoint C .

The force F_x equals the inertia force of the part of the rotating bar from D to B .

Consider an element of mass dM at distance ξ from the midpoint C . The variable ξ ranges from x to L .

$$dM = \frac{\gamma}{g} A d\xi$$

dF = Inertia force (centrifugal force) of element of mass dM

$$dF = (dM)(\xi\omega^2) = \frac{\gamma}{g} A\omega^2 \xi d\xi$$

$$F_x = \int_D^B dF = \int_x^L \frac{\gamma}{g} A\omega^2 \xi d\xi = \frac{\gamma A \omega^2}{2g} (L^2 - x^2)$$

(a) TENSILE STRESS IN BAR AT DISTANCE x

$$\sigma_x = \frac{F_x}{A} = \frac{\gamma\omega^2}{2g} (L^2 - x^2) \longleftarrow$$

(b) MAXIMUM TENSILE STRESS

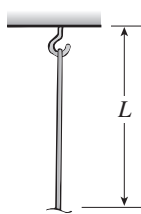
$$x = 0 \quad \sigma_{\max} = \frac{\gamma\omega^2 L^2}{2g} \longleftarrow$$

Mechanical Properties and Stress-Strain Diagrams

Problem 1.3-1 Imagine that a long steel wire hangs vertically from a high-altitude balloon.

- (a) What is the greatest length (feet) it can have without yielding if the steel yields at 40 ksi?
- (b) If the same wire hangs from a ship at sea, what is the greatest length? (Obtain the weight densities of steel and sea water from Table H-1, Appendix H.)

Solution 1.3-1 Hanging wire of length L



W = total weight of steel wire

γ_s = weight density of steel
= 490 lb/ft³

γ_w = weight density of sea water
= 63.8 lb/ft³

A = cross-sectional area of wire

σ_{\max} = 40 psi (yield strength)

(a) WIRE HANGING IN AIR

$$W = \gamma_s AL$$

$$\sigma_{\max} = \frac{W}{A} = \gamma_s L$$

$$L_{\max} = \frac{\sigma_{\max}}{\gamma_s} = \frac{40,000 \text{ psi}}{490 \text{ lb/ft}^3} (144 \text{ in.}^2/\text{ft}^2) = 11,800 \text{ ft} \longleftarrow$$

(b) WIRE HANGING IN SEA WATER

F = tensile force at top of wire

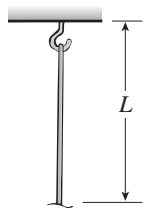
$$F = (\gamma_s - \gamma_w) AL \quad \sigma_{\max} = \frac{F}{A} = (\gamma_s - \gamma_w)L$$

$$L_{\max} = \frac{\sigma_{\max}}{\gamma_s - \gamma_w} = \frac{40,000 \text{ psi}}{(490 - 63.8) \text{ lb/ft}^3} (144 \text{ in.}^2/\text{ft}^2) = 13,500 \text{ ft} \longleftarrow$$

Problem 1.3-2 Imagine that a long wire of tungsten hangs vertically from a high-altitude balloon.

- What is the greatest length (meters) it can have without breaking if the ultimate strength (or breaking strength) is 1500 MPa?
- If the same wire hangs from a ship at sea, what is the greatest length? (Obtain the weight densities of tungsten and sea water from Table H-1, Appendix H.)

Solution 1.3-2 Hanging wire of length L



W = total weight of tungsten wire

$$\gamma_T = \text{weight density of tungsten} \\ = 190 \text{ kN/m}^3$$

$$\gamma_W = \text{weight density of sea water} \\ = 10.0 \text{ kN/m}^3$$

A = cross-sectional area of wire

$$\sigma_{\max} = 1500 \text{ MPa (breaking strength)}$$

(a) WIRE HANGING IN AIR

$$W = \gamma_T AL$$

$$\sigma_{\max} = \frac{W}{A} = \gamma_T L$$

$$L_{\max} = \frac{\sigma_{\max}}{\gamma_T} = \frac{1500 \text{ MPa}}{190 \text{ kN/m}^3}$$

$$7900 \text{ m} \leftarrow$$

(b) WIRE HANGING IN SEA WATER

F = tensile force at top of wire

$$F = (\gamma_T - \gamma_W)AL$$

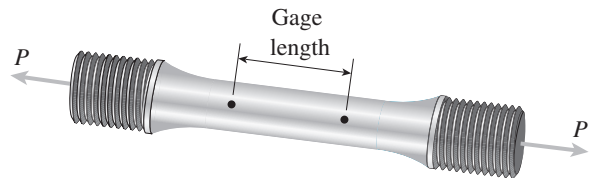
$$\sigma_{\max} = \frac{F}{A} = (\gamma_T - \gamma_W)L$$

$$L_{\max} = \frac{\sigma_{\max}}{\gamma_T - \gamma_W} \\ = \frac{1500 \text{ MPa}}{(190 - 10.0) \text{ kN/m}^3}$$

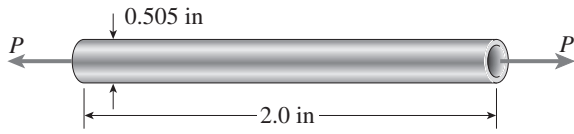
$$= 8300 \text{ m} \leftarrow$$

Problem 1.3-3 Three different materials, designated A , B , and C , are tested in tension using test specimens having diameters of 0.505 in. and gage lengths of 2.0 in. (see figure). At failure, the distances between the gage marks are found to be 2.13, 2.48, and 2.78 in., respectively. Also, at the failure cross sections the diameters are found to be 0.484, 0.398, and 0.253 in., respectively.

Determine the percent elongation and percent reduction in area of each specimen, and then, using your own judgment, classify each material as brittle or ductile.



Solution 1.3-3 Tensile tests of three materials



$$\text{Percent elongation} = \frac{L_1 - L_0}{L_0} (100) = \left(\frac{L_1}{L_0} - 1 \right) 100$$

$$L_0 = 2.0 \text{ in.}$$

$$\text{Percent elongation} = \left(\frac{L_1}{2.0} - 1 \right) (100) \quad (\text{Eq. 1})$$

where L_1 is in inches.

$$\begin{aligned} \text{Percent reduction in area} &= \frac{A_0 - A_1}{A_0} (100) \\ &= \left(1 - \frac{A_1}{A_0} \right) (100) \end{aligned}$$

d_0 = initial diameter d_1 = final diameter

$$\frac{A_1}{A_0} = \left(\frac{d_1}{d_0} \right)^2 \quad d_0 = 0.505 \text{ in.}$$

$$\begin{aligned} \text{Percent reduction in area} \\ &= \left[1 - \left(\frac{d_1}{0.505} \right)^2 \right] (100) \quad (\text{Eq. 2}) \end{aligned}$$

where d_1 is in inches.

Material	L_1 (in.)	d_1 (in.)	% Elongation (Eq. 1)	% Reduction (Eq. 2)	Brittle or Ductile?
A	2.13	0.484	6.5%	8.1%	Brittle
B	2.48	0.398	24.0%	37.9%	Ductile
C	2.78	0.253	39.0%	74.9%	Ductile

Problem 1.3-4 The *strength-to-weight ratio* of a structural material is defined as its load-carrying capacity divided by its weight. For materials in tension, we may use a characteristic tensile stress (as obtained from a stress-strain curve) as a measure of strength. For instance, either the yield stress or the ultimate stress could be used, depending upon the particular application. Thus, the strength-to-weight ratio $R_{S/W}$ for a material in tension is defined as

$$R_{S/W} = \frac{\sigma}{\gamma}$$

in which σ is the characteristic stress and γ is the weight density. Note that the ratio has units of length. Using the ultimate stress σ_u as the strength parameter, calculate the strength-to-weight ratio (in units of meters) for each of the following materials: aluminum alloy 6061-T6, Douglas fir (in bending), nylon, structural steel ASTM-A572, and a titanium alloy. (Obtain the material properties from Tables H-1 and H-3 of Appendix H. When a range of values is given in a table, use the average value.)

Solution 1.3-4 Strength-to-weight ratio

The ultimate stress σ_u for each material is obtained from Table H-3, Appendix H, and the weight density γ is obtained from Table H-1.

The strength-to-weight ratio (meters) is

$$R_{S/W} = \frac{\sigma_u (\text{MPa})}{\gamma (\text{kN/m}^3)} (10^3)$$

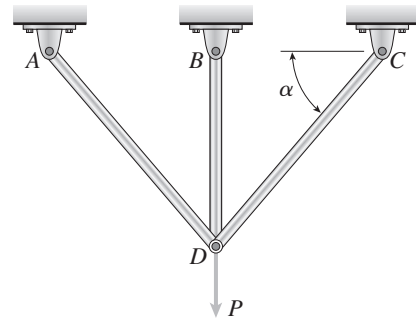
Values of σ_u , γ , and $R_{S/W}$ are listed in the table.

	σ_u (MPa)	γ (kN/m ³)	$R_{S/W}$ (m)
Aluminum alloy 6061-T6	310	26.0	11.9×10^3
Douglas fir	65	5.1	12.7×10^3
Nylon	60	9.8	6.1×10^3
Structural steel ASTM-A572	500	77.0	6.5×10^3
Titanium alloy	1050	44.0	23.9×10^3

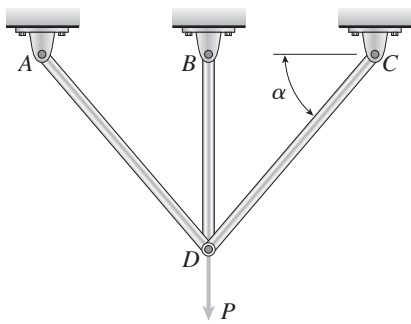
Titanium has a high strength-to-weight ratio, which is why it is used in space vehicles and high-performance airplanes. Aluminum is higher than steel, which makes it desirable for commercial aircraft. Some woods are also higher than steel, and nylon is about the same as steel.

Problem 1.3-5 A symmetrical framework consisting of three pin-connected bars is loaded by a force P (see figure). The angle between the inclined bars and the horizontal is $\alpha = 48^\circ$. The axial strain in the middle bar is measured as 0.0713.

Determine the tensile stress in the outer bars if they are constructed of aluminum alloy having the stress-strain diagram shown in Fig. 1-13. (Express the stress in USCS units.)



Solution 1.3-5 Symmetrical framework

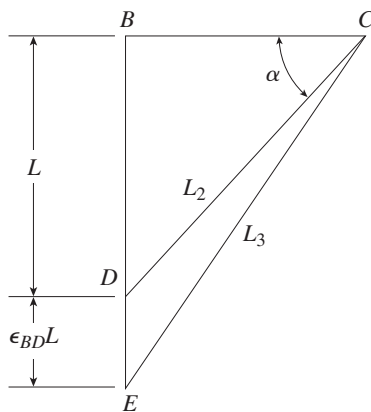


Aluminum alloy

$$\alpha = 48^\circ$$

$$\epsilon_{BD} = 0.0713$$

Use stress-strain diagram of Figure 1-13



L = length of bar BD

L_1 = distance BC

$$= L \cot \alpha = L(\cot 48^\circ) = 0.9004 L$$

L_2 = length of bar CD

$$= L \csc \alpha = L(\csc 48^\circ) = 1.3456 L$$

Elongation of bar BD = distance $DE = \epsilon_{BD}L$

$$\epsilon_{BD}L = 0.0713 L$$

L_3 = distance CE

$$\begin{aligned} L_3 &= \sqrt{L_1^2 + (L + \epsilon_{BD}L)^2} \\ &= \sqrt{(0.9004L)^2 + L^2(1 + 0.0713)^2} \\ &= 1.3994 L \end{aligned}$$

δ = elongation of bar CD

$$\delta = L_3 - L_2 = 0.0538L$$

Strain in bar CD

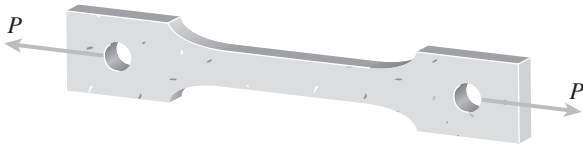
$$= \frac{\delta}{L_2} = \frac{0.0538L}{1.3456L} = 0.0400$$

From the stress-strain diagram of Figure 1-13:

$$\sigma \approx 31 \text{ ksi}$$

Problem 1.3-6 A specimen of a methacrylate plastic is tested in tension at room temperature (see figure), producing the stress-strain data listed in the accompanying table.

Plot the stress-strain curve and determine the proportional limit, modulus of elasticity (i.e., the slope of the initial part of the stress-strain curve), and yield stress at 0.2% offset. Is the material ductile or brittle?

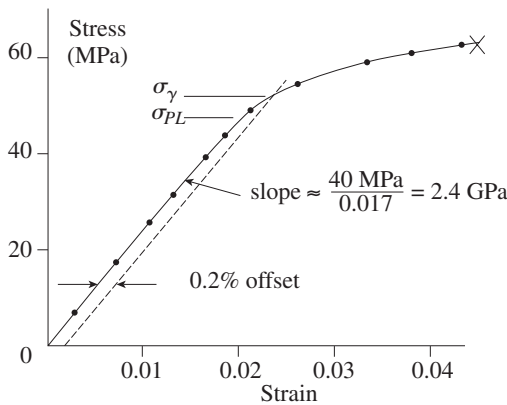


STRESS-STRAIN DATA FOR PROBLEM 1.3-6

Stress (MPa)	Strain
8.0	0.0032
17.5	0.0073
25.6	0.0111
31.1	0.0129
39.8	0.0163
44.0	0.0184
48.2	0.0209
53.9	0.0260
58.1	0.0331
62.0	0.0429
62.1	Fracture

Solution 1.3-6 Tensile test of a plastic

Using the stress-strain data given in the problem statement, plot the stress-strain curve:



σ_{PL} = proportional limit $\sigma_{PL} \approx 47$ MPa ←
 Modulus of elasticity (slope) ≈ 2.4 GPa ←
 σ_Y = yield stress at 0.2% offset
 $\sigma_Y \approx 53$ MPa ←
 Material is *brittle*, because the strain after the proportional limit is exceeded is relatively small. ←

Problem 1.3-7 The data shown in the accompanying table were obtained from a tensile test of high-strength steel. The test specimen had a diameter of 0.505 in. and a gage length of 2.00 in. (see figure for Prob. 1.3-3). At fracture, the elongation between the gage marks was 0.12 in. and the minimum diameter was 0.42 in.

Plot the conventional stress-strain curve for the steel and determine the proportional limit, modulus of elasticity (i.e., the slope of the initial part of the stress-strain curve), yield stress at 0.1% offset, ultimate stress, percent elongation in 2.00 in., and percent reduction in area.

TENSILE-TEST DATA FOR PROBLEM 1.3-7

Load (lb)	Elongation (in.)
1,000	0.0002
2,000	0.0006
6,000	0.0019
10,000	0.0033
12,000	0.0039
12,900	0.0043
13,400	0.0047
13,600	0.0054
13,800	0.0063
14,000	0.0090
14,400	0.0102
15,200	0.0130
16,800	0.0230
18,400	0.0336
20,000	0.0507
22,400	0.1108
22,600	Fracture

Solution 1.3-7 Tensile test of high-strength steel

$$d_0 = 0.505 \text{ in.} \quad L_0 = 2.00 \text{ in.}$$

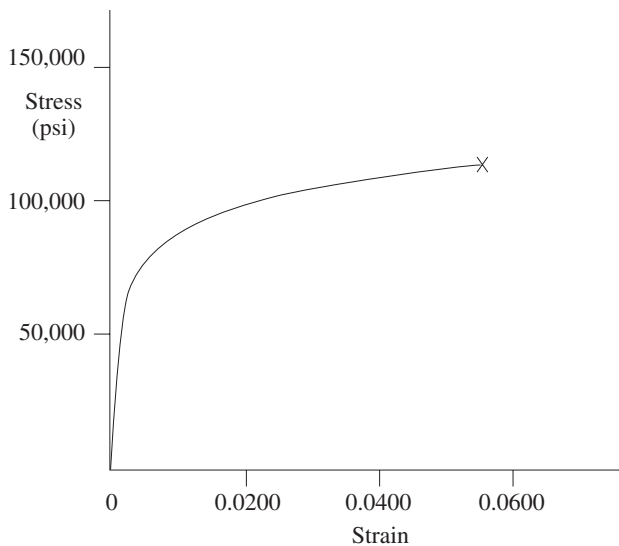
$$A_0 = \frac{\pi d_0^2}{4} = 0.200 \text{ in.}^2$$

CONVENTIONAL STRESS AND STRAIN

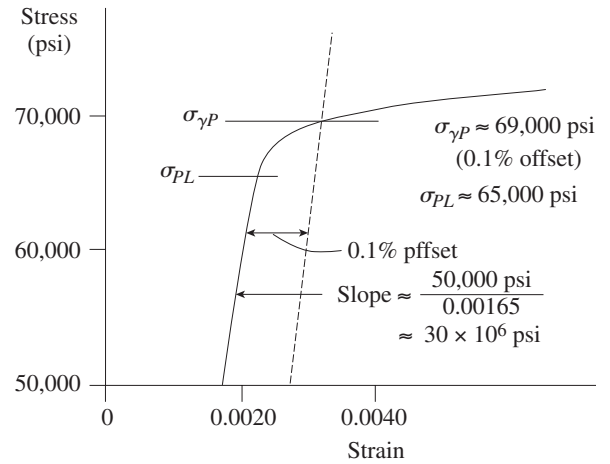
$$\sigma = \frac{P}{A_0} \quad \varepsilon = \frac{\delta}{L_0}$$

Load P (lb)	Elongation δ (in.)	Stress σ (psi)	Strain ε
1,000	0.0002	5,000	0.00010
2,000	0.0006	10,000	0.00030
6,000	0.0019	30,000	0.00100
10,000	0.0033	50,000	0.00165
12,000	0.0039	60,000	0.00195
12,900	0.0043	64,500	0.00215
13,400	0.0047	67,000	0.00235
13,600	0.0054	68,000	0.00270
13,800	0.0063	69,000	0.00315
14,000	0.0090	70,000	0.00450
14,400	0.0102	72,000	0.00510
15,200	0.0130	76,000	0.00650
16,800	0.0230	84,000	0.01150
18,400	0.0336	92,000	0.01680
20,000	0.0507	100,000	0.02535
22,400	0.1108	112,000	0.05540
22,600	Fracture	113,000	

STRESS-STRAIN DIAGRAM



ENLARGEMENT OF PART OF THE STRESS-STRAIN CURVE



RESULTS

Proportional limit $\approx 65,000$ psi \leftarrow

Modulus of elasticity (slope) $\approx 30 \times 10^6$ psi \leftarrow

Yield stress at 0.1% offset $\approx 69,000$ psi \leftarrow

Ultimate stress (maximum stress)

$\approx 113,000$ psi \leftarrow

Percent elongation in 2.00 in.

$$= \frac{L_1 - L_0}{L_0} (100)$$

$$= \frac{0.12 \text{ in.}}{2.00 \text{ in.}} (100) = 6\% \leftarrow$$

Percent reduction in area

$$= \frac{A_0 - A_1}{A_0} (100)$$

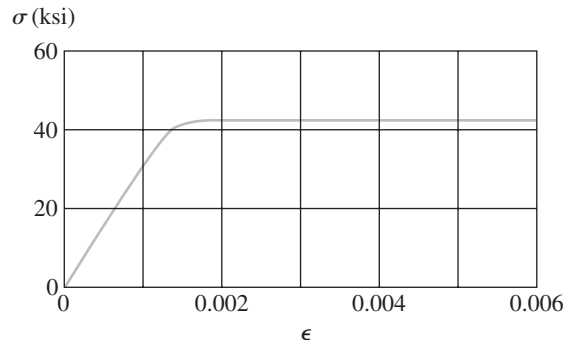
$$= \frac{0.200 \text{ in.}^2 - \frac{\pi}{4}(0.42 \text{ in.})^2}{0.200 \text{ in.}^2} (100)$$

$$= 31\% \leftarrow$$

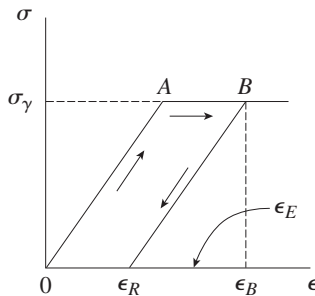
Elasticity and Plasticity

Problem 1.4-1 A bar made of structural steel having the stress-strain diagram shown in the figure has a length of 48 in. The yield stress of the steel is 42 ksi and the slope of the initial linear part of the stress-strain curve (modulus of elasticity) is 30×10^3 ksi. The bar is loaded axially until it elongates 0.20 in., and then the load is removed.

How does the final length of the bar compare with its original length? (*Hint*: Use the concepts illustrated in Fig. 1-18b.)



Solution 1.4-1 Steel bar in tension



$L = 48$ in.

Yield stress $\sigma_Y = 42$ ksi

Slope = 30×10^3 ksi

$\delta = 0.20$ in.

STRESS AND STRAIN AT POINT *B*

$\sigma_B = \sigma_Y = 42$ ksi

$\epsilon_B = \frac{\delta}{L} = \frac{0.20 \text{ in.}}{48 \text{ in.}} = 0.00417$

ELASTIC RECOVERY ϵ_E

$\epsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{42 \text{ ksi}}{30 \times 10^3 \text{ ksi}} = 0.00140$

RESIDUAL STRAIN ϵ_R

$\epsilon_R = \epsilon_B - \epsilon_E = 0.00417 - 0.00140 = 0.00277$

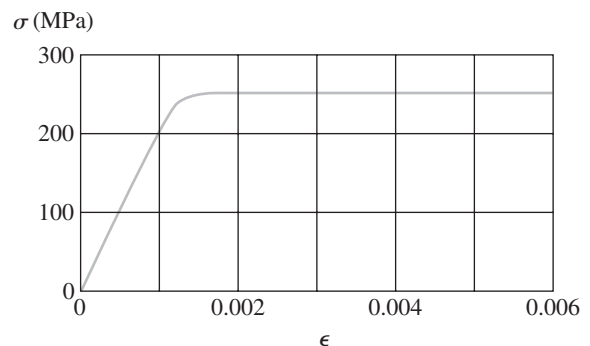
PERMANENT SET

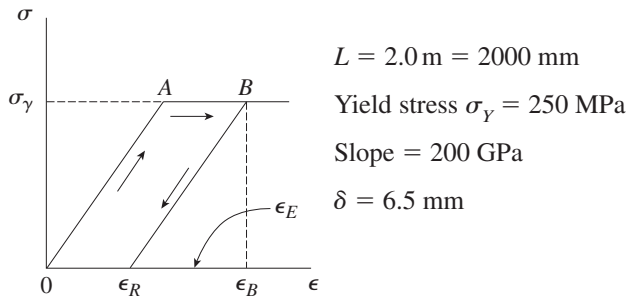
$\epsilon_R L = (0.00277)(48 \text{ in.}) = 0.13 \text{ in.}$

Final length of bar is 0.13 in. greater than its original length. ←

Problem 1.4-2 A bar of length 2.0 m is made of a structural steel having the stress-strain diagram shown in the figure. The yield stress of the steel is 250 MPa and the slope of the initial linear part of the stress-strain curve (modulus of elasticity) is 200 GPa. The bar is loaded axially until it elongates 6.5 mm, and then the load is removed.

How does the final length of the bar compare with its original length? (*Hint*: Use the concepts illustrated in Fig. 1-18b.)



Solution 1.4-2 Steel bar in tensionSTRESS AND STRAIN AT POINT *B*

$$\sigma_B = \sigma_Y = 250 \text{ MPa}$$

$$\varepsilon_B = \frac{\delta}{L} = \frac{6.5 \text{ mm}}{2000 \text{ mm}} = 0.00325$$

ELASTIC RECOVERY ε_E

$$\varepsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{250 \text{ MPa}}{200 \text{ GPa}} = 0.00125$$

RESIDUAL STRAIN ε_R

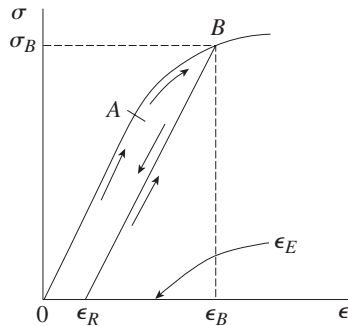
$$\varepsilon_R = \varepsilon_B - \varepsilon_E = 0.00325 - 0.00125 = 0.00200$$

$$\begin{aligned} \text{Permanent set} &= \varepsilon_R L = (0.00200)(2000 \text{ mm}) \\ &= 4.0 \text{ mm} \end{aligned}$$

Final length of bar is 4.0 mm greater than its original length. ←

Problem 1.4-3 An aluminum bar has length $L = 4 \text{ ft}$ and diameter $d = 1.0 \text{ in.}$ The stress-strain curve for the aluminum is shown in Fig. 1-13 of Section 1.3. The initial straight-line part of the curve has a slope (modulus of elasticity) of $10 \times 10^6 \text{ psi}$. The bar is loaded by tensile forces $P = 24 \text{ k}$ and then unloaded.

- (a) What is the permanent set of the bar?
 (b) If the bar is reloaded, what is the proportional limit?
 (*Hint:* Use the concepts illustrated in Figs. 1-18b and 1-19.)

Solution 1.4-3 Aluminum bar in tension

$$L = 4 \text{ ft} = 48 \text{ in.}$$

$$d = 1.0 \text{ in.}$$

$$P = 24 \text{ k}$$

See Fig. 1-13 for stress-strain diagram

Slope from *O* to *A* is $10 \times 10^6 \text{ psi}$.STRESS AND STRAIN AT POINT *B*

$$\sigma_B = \frac{P}{A} = \frac{24 \text{ k}}{\frac{\pi}{4}(1.0 \text{ in.})^2} = 31 \text{ ksi}$$

$$\text{From Fig. 1-13: } \varepsilon_B \approx 0.04$$

ELASTIC RECOVERY ε_E

$$\varepsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{31 \text{ ksi}}{10 \times 10^6 \text{ psi}} = 0.0031$$

RESIDUAL STRAIN ε_R

$$\varepsilon_R = \varepsilon_B - \varepsilon_E = 0.04 - 0.0031 = 0.037$$

(Note: the accuracy in this result is very poor because ε_B is approximate.)

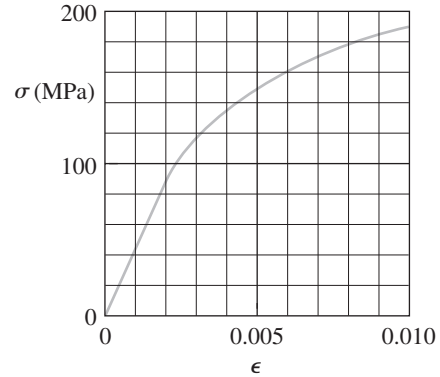
(a) PERMANENT SET

$$\begin{aligned} \varepsilon_R L &= (0.037)(48 \text{ in.}) \\ &\approx 1.8 \text{ in.} \quad \leftarrow \end{aligned}$$

(b) PROPORTIONAL LIMIT WHEN RELOADED

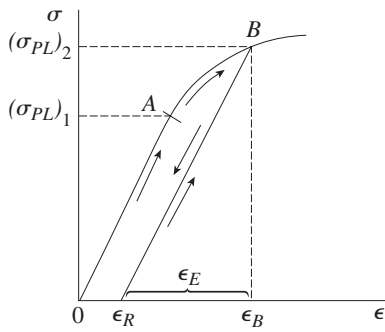
$$\sigma_B = 31 \text{ ksi} \quad \leftarrow$$

Problem 1.4-4 A circular bar of magnesium alloy is 800 mm long. The stress-strain diagram for the material is shown in the figure. The bar is loaded in tension to an elongation of 5.6 mm, and then the load is removed.



- (a) What is the permanent set of the bar?
- (b) If the bar is reloaded, what is the proportional limit?
(Hint: Use the concepts illustrated in Figs. 1-18b and 1-19.)

Solution 1.4-4 Magnesium bar in tension



$L = 800 \text{ mm}$
 $\delta = 5.6 \text{ mm}$

$(\sigma_{PL})_1 =$ initial proportional limit
 $= 88 \text{ MPa}$ (from stress-strain diagram)
 $(\sigma_{PL})_2 =$ proportional limit when the bar is reloaded

INITIAL SLOPE OF STRESS-STRAIN CURVE
 From $\sigma - \epsilon$ diagram:

At point A: $(\sigma_{PL})_1 = 88 \text{ MPa}$
 $\epsilon_A = 0.002$

$$\text{Slope} = \frac{(\sigma_{pl})_1}{\epsilon_A} = \frac{88 \text{ MPa}}{0.002} = 44 \text{ GPa}$$

STRESS AND STRAIN AT POINT B

$$\epsilon_B = \frac{\delta}{L} = \frac{5.6 \text{ mm}}{800 \text{ mm}} = 0.007$$

From $\sigma - \epsilon$ diagram: $\sigma_B = (\sigma_{PL})_2 = 170 \text{ MPa}$

ELASTIC RECOVERY ϵ_E

$$\epsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{(\sigma_{PL})_2}{\text{Slope}} = \frac{170 \text{ MPa}}{44 \text{ GPa}} = 0.00386$$

RESIDUAL STRAIN ϵ_R

$$\epsilon_R = \epsilon_B - \epsilon_E = 0.007 - 0.00386 = 0.00314$$

(a) PERMANENT SET

$$\epsilon_R L = (0.00314)(800 \text{ mm}) = 2.51 \text{ mm} \leftarrow$$

(b) PROPORTIONAL LIMIT WHEN RELOADED

$$(\sigma_{PL})_2 = \sigma_B = 170 \text{ MPa} \leftarrow$$

Problem 1.4-5 A wire of length $L = 4 \text{ ft}$ and diameter $d = 0.125 \text{ in.}$ is stretched by tensile forces $P = 600 \text{ lb.}$ The wire is made of a copper alloy having a stress-strain relationship that may be described mathematically by the following equation:

$$\sigma = \frac{18,000\epsilon}{1 + 300\epsilon} \quad 0 \leq \epsilon \leq 0.03 \quad (\sigma = \text{ksi})$$

in which ϵ is nondimensional and σ has units of kips per square inch (ksi).

- (a) Construct a stress-strain diagram for the material.
- (b) Determine the elongation of the wire due to the forces P .
- (c) If the forces are removed, what is the permanent set of the bar?
- (d) If the forces are applied again, what is the proportional limit?

Solution 1.4-5 Wire stretched by forces P

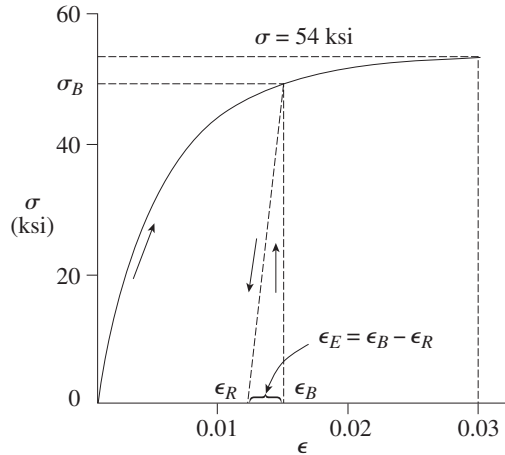
$$L = 4 \text{ ft} = 48 \text{ in.} \quad d = 0.125 \text{ in.}$$

$$P = 600 \text{ lb}$$

COPPER ALLOY

$$\sigma = \frac{18,000\varepsilon}{1 + 300\varepsilon} \quad 0 \leq \varepsilon \leq 0.03 \quad (\sigma = \text{ksi}) \quad (\text{Eq. 1})$$

(a) STRESS-STRAIN DIAGRAM (From Eq. 1)



INITIAL SLOPE OF STRESS-STRAIN CURVE

Take the derivative of σ with respect to ε :

$$\begin{aligned} \frac{d\sigma}{d\varepsilon} &= \frac{(1 + 300\varepsilon)(18,000) - (18,000\varepsilon)(300)}{(1 + 300\varepsilon)^2} \\ &= \frac{18,000}{(1 + 300\varepsilon)^2} \end{aligned}$$

$$\text{At } \varepsilon = 0, \quad \frac{d\sigma}{d\varepsilon} = 18,000 \text{ ksi}$$

$$\therefore \text{Initial slope} = 18,000 \text{ ksi}$$

ALTERNATIVE FORM OF THE STRESS-STRAIN RELATIONSHIP

Solve Eq. (1) for ε in terms of σ :

$$\varepsilon = \frac{\sigma}{18,000 - 300\sigma} \quad 0 \leq \sigma \leq 54 \text{ ksi} \quad (\sigma = \text{ksi}) \quad (\text{Eq. 2})$$

This equation may also be used when plotting the stress-strain diagram.

(b) ELONGATION δ OF THE WIRE

$$\sigma = \frac{P}{A} = \frac{600 \text{ lb}}{\frac{\pi}{4}(0.125 \text{ in.})^2} = 48,900 \text{ psi} = 48.9 \text{ ksi}$$

From Eq. (2) or from the stress-strain diagram:

$$\varepsilon = 0.0147$$

$$\delta = \varepsilon L = (0.0147)(48 \text{ in.}) = 0.71 \text{ in.} \quad \leftarrow$$

STRESS AND STRAIN AT POINT B (see diagram)

$$\sigma_B = 48.9 \text{ ksi} \quad \varepsilon_B = 0.0147$$

ELASTIC RECOVERY ε_E

$$\varepsilon_E = \frac{\sigma_B}{\text{Slope}} = \frac{48.9 \text{ ksi}}{18,000 \text{ ksi}} = 0.00272$$

RESIDUAL STRAIN ε_R

$$\varepsilon_R = \varepsilon_B - \varepsilon_E = 0.0147 - 0.0027 = 0.0120$$

$$\begin{aligned} \text{(c) Permanent set} &= \varepsilon_R L = (0.0120)(48 \text{ in.}) \\ &= 0.58 \text{ in.} \quad \leftarrow \end{aligned}$$

(d) Proportional limit when reloaded = σ_B

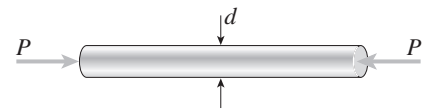
$$\sigma_B = 49 \text{ ksi} \quad \leftarrow$$

Hooke's Law and Poisson's Ratio

When solving the problems for Section 1.5, assume that the material behaves linearly elastically.

Problem 1.5-1 A high-strength steel bar used in a large crane has diameter $d = 2.00$ in. (see figure). The steel has modulus of elasticity $E = 29 \times 10^6$ psi and Poisson's ratio $\nu = 0.29$. Because of clearance requirements, the diameter of the bar is limited to 2.001 in. when it is compressed by axial forces.

What is the largest compressive load P_{\max} that is permitted?



Solution 1.5-1 Steel bar in compression

STEEL BAR $d = 2.00$ in. Max. $\Delta d = 0.001$ in.
 $E = 29 \times 10^6$ psi $\nu = 0.29$

LATERAL STRAIN

$$\varepsilon' = \frac{\Delta d}{d} = \frac{0.001 \text{ in.}}{2.00 \text{ in.}} = 0.0005$$

AXIAL STRAIN

$$\varepsilon = -\frac{\varepsilon'}{\nu} = -\frac{0.0005}{0.29} = -0.001724$$

(shortening)

AXIAL STRESS

$$\sigma = E\varepsilon = (29 \times 10^6 \text{ psi})(-0.001724) \\ = -50.00 \text{ ksi (compression)}$$

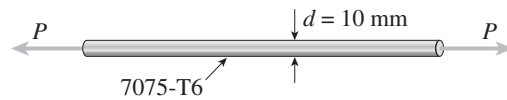
Assume that the yield stress for the high-strength steel is greater than 50 ksi. Therefore, Hooke's law is valid.

MAXIMUM COMPRESSIVE LOAD

$$P_{\max} = \sigma A = (50.00 \text{ ksi})\left(\frac{\pi}{4}\right)(2.00 \text{ in.})^2 \\ = 157 \text{ k} \quad \leftarrow$$

Problem 1.5-2 A round bar of 10 mm diameter is made of aluminum alloy 7075-T6 (see figure). When the bar is stretched by axial forces P , its diameter decreases by 0.016 mm.

Find the magnitude of the load P . (Obtain the material properties from Appendix H.)

**Solution 1.5-2 Aluminum bar in tension**

$d = 10$ mm $\Delta d = 0.016$ mm
 (Decrease in diameter)

7075-T6

From Table H-2: $E = 72$ GPa $\nu = 0.33$

From Table H-3: Yield stress $\sigma_Y = 480$ MPa

LATERAL STRAIN

$$\varepsilon' = \frac{\Delta d}{d} = \frac{-0.016 \text{ mm}}{10 \text{ mm}} = -0.0016$$

AXIAL STRAIN

$$\varepsilon = -\frac{-\varepsilon'}{\nu} = \frac{0.0016}{0.33} \\ = 0.004848 \text{ (Elongation)}$$

AXIAL STRESS

$$\sigma = E\varepsilon = (72 \text{ GPa})(0.004848) \\ = 349.1 \text{ MPa (Tension)}$$

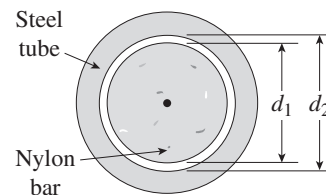
Because $\sigma < \sigma_Y$, Hooke's law is valid.

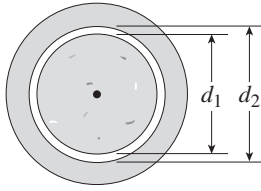
LOAD P (TENSILE FORCE)

$$P = \sigma A = (349.1 \text{ MPa})\left(\frac{\pi}{4}\right)(10 \text{ mm})^2 \\ = 27.4 \text{ kN} \quad \leftarrow$$

Problem 1.5-3 A nylon bar having diameter $d_1 = 3.50$ in. is placed inside a steel tube having inner diameter $d_2 = 3.51$ in. (see figure). The nylon bar is then compressed by an axial force P .

At what value of the force P will the space between the nylon bar and the steel tube be closed? (For nylon, assume $E = 400$ ksi and $\nu = 0.4$.)



Solution 1.5-3 Nylon bar inside steel tube

Compression

$$d_1 = 3.50 \text{ in.} \quad \Delta d_1 = 0.01 \text{ in.}$$

$$d_2 = 3.51 \text{ in.}$$

$$\text{Nylon: } E = 400 \text{ ksi} \quad \nu = 0.4$$

LATERAL STRAIN

$$\varepsilon' = \frac{\Delta d_1}{d_1} \text{ (Increase in diameter)}$$

$$\varepsilon' = \frac{0.01 \text{ in.}}{3.50 \text{ in.}} = 0.002857$$

AXIAL STRAIN

$$\varepsilon = -\frac{\varepsilon'}{\nu} = -\frac{0.002857}{0.4} = -0.007143$$

(Shortening)

AXIAL STRESS

$$\sigma = E\varepsilon = (400 \text{ ksi})(-0.007143) \\ = -2.857 \text{ ksi}$$

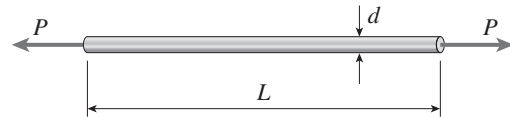
(Compressive stress)

Assume that the yield stress is greater than σ and Hooke's law is valid.FORCE P (COMPRESSION)

$$P = \sigma A = (2.857 \text{ ksi})\left(\frac{\pi}{4}\right)(3.50 \text{ in.})^2 \\ = 27.5 \text{ k} \quad \longleftarrow$$

Problem 1.5-4 A prismatic bar of circular cross section is loaded by tensile forces P (see figure). The bar has length $L = 1.5 \text{ m}$ and diameter $d = 30 \text{ mm}$. It is made of aluminum alloy with modulus of elasticity $E = 75 \text{ GPa}$ and Poisson's ratio $\nu = \frac{1}{3}$.

If the bar elongates by 3.6 mm , what is the decrease in diameter Δd ? What is the magnitude of the load P ?

**Solution 1.5-4 Aluminum bar in tension**

$$L = 1.5 \text{ m} \quad d = 30 \text{ mm}$$

$$E = 75 \text{ GPa} \quad \nu = \frac{1}{3}$$

$$\delta = 3.6 \text{ mm (elongation)}$$

AXIAL STRAIN

$$\varepsilon = \frac{\delta}{L} = \frac{3.6 \text{ mm}}{1.5 \text{ m}} = 0.0024$$

LATERAL STRAIN

$$\varepsilon' = -\nu\varepsilon = -\left(\frac{1}{3}\right)(0.0024) \\ = -0.0008$$

(Minus means decrease in diameter)

DECREASE IN DIAMETER

$$\Delta d = \varepsilon' d = (0.0008)(30 \text{ mm}) = 0.024 \text{ mm} \quad \longleftarrow$$

AXIAL STRESS

$$\sigma = E\varepsilon = (75 \text{ GPa})(0.0024) \\ = 180 \text{ MPa}$$

(This stress is less than the yield stress, so Hooke's law is valid.)

LOAD P (TENSION)

$$P = \sigma A = (180 \text{ MPa})\left(\frac{\pi}{4}\right)(30 \text{ mm})^2 \\ = 127 \text{ kN} \quad \longleftarrow$$

Problem 1.5-5 A bar of monel metal (length $L = 8$ in., diameter $d = 0.25$ in.) is loaded axially by a tensile force $P = 1500$ lb (see figure). Using the data in Table H-2,

Appendix H, determine the increase in length of the bar and the percent decrease in its cross-sectional area.

Solution 1.5-5 Bar of monel metal in tension

$L = 8$ in. $d = 0.25$ in. $P = 1500$ lb

From Table H-2: $E = 25,000$ ksi $\nu = 0.32$

AXIAL STRESS

$$\sigma = \frac{P}{A} = \frac{1500 \text{ lb}}{\frac{\pi}{4}(0.25 \text{ in.})^2} = 30,560 \text{ psi}$$

Assume σ is less than the proportional limit, so that Hooke's law is valid.

AXIAL STRAIN

$$\varepsilon = \frac{\sigma}{E} = \frac{30,560 \text{ psi}}{25,000 \text{ ksi}} = 0.001222$$

INCREASE IN LENGTH

$$\delta = \varepsilon L = (0.001222)(8 \text{ in.}) = 0.00978 \text{ in.} \quad \leftarrow$$

LATERAL STRAIN

$$\begin{aligned} \varepsilon' &= -\nu\varepsilon = -(0.32)(0.001222) \\ &= -0.0003910 \end{aligned}$$

DECREASE IN DIAMETER

$$\begin{aligned} \Delta d &= |\varepsilon' d| = (0.0003910)(0.25 \text{ in.}) \\ &= 0.0000978 \text{ in.} \end{aligned}$$

DECREASE IN CROSS-SECTIONAL AREA

Original area: $A_0 = \frac{\pi d^2}{4}$

Final area:

$$A_1 = \frac{\pi}{4}(d - \Delta d)^2$$

$$A_1 = \frac{\pi}{4}[d^2 - 2d\Delta d + (\Delta d)^2]$$

Decrease in area:

$$\Delta A = A_0 - A_1$$

$$\Delta A = \frac{\pi}{4}(\Delta d)(2d - \Delta d)$$

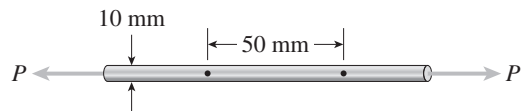
PERCENT DECREASE IN AREA

$$\text{Percent} = \frac{\Delta A}{A_0} (100) = \frac{(\Delta d)(2d - \Delta d)}{d^2} (100)$$

$$= \frac{(0.0000978)(0.4999)}{(0.25)^2} (100)$$

$$= 0.078\% \quad \leftarrow$$

Problem 1.5-6 A tensile test is performed on a brass specimen 10 mm in diameter using a gage length of 50 mm (see figure). When the tensile load P reaches a value of 20 kN, the distance between the gage marks has increased by 0.122 mm.



- (a) What is the modulus of elasticity E of the brass?
- (b) If the diameter decreases by 0.00830 mm, what is Poisson's ratio?

Solution 1.5-6 Brass specimen in tension

$$d = 10 \text{ mm} \quad \text{Gage length } L = 50 \text{ mm}$$

$$P = 20 \text{ kN} \quad \delta = 0.122 \text{ mm} \quad \Delta d = 0.00830 \text{ mm}$$

AXIAL STRESS

$$\sigma = \frac{P}{A} = \frac{20 \text{ kN}}{\frac{\pi}{4}(10 \text{ mm})^2} = 254.6 \text{ MPa}$$

Assume σ is below the proportional limit so that Hooke's law is valid.

AXIAL STRAIN

$$\varepsilon = \frac{\delta}{L} = \frac{0.122 \text{ mm}}{50 \text{ mm}} = 0.002440$$

(a) MODULUS OF ELASTICITY

$$E = \frac{\sigma}{\varepsilon} = \frac{254.6 \text{ MPa}}{0.002440} = 104 \text{ GPa} \leftarrow$$

(b) POISSON'S RATIO

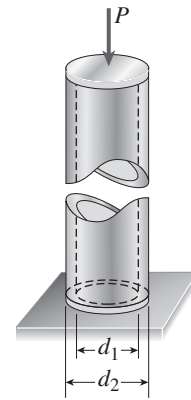
$$\varepsilon' = \nu \varepsilon$$

$$\Delta d = \varepsilon' d = \nu \varepsilon d$$

$$\nu = \frac{\Delta d}{\varepsilon d} = \frac{0.00830 \text{ mm}}{(0.002440)(10 \text{ mm})} = 0.34 \leftarrow$$

Problem 1.5-7 A hollow steel cylinder is compressed by a force P (see figure). The cylinder has inner diameter $d_1 = 3.9$ in., outer diameter $d_2 = 4.5$ in., and modulus of elasticity $E = 30,000$ ksi. When the force P increases from zero to 40 k, the outer diameter of the cylinder increases by 455×10^{-6} in.

- Determine the increase in the inner diameter.
- Determine the increase in the wall thickness.
- Determine Poisson's ratio for the steel.

**Solution 1.5-7 Hollow steel cylinder**

$$d_1 = 3.9 \text{ in.}$$

$$d_2 = 4.5 \text{ in.}$$

$$t = 0.3 \text{ in.}$$

$$E = 30,000 \text{ ksi}$$

$$P = 40 \text{ k (compression)}$$

$$\Delta d_2 = 455 \times 10^{-6} \text{ in. (increase)}$$

LATERAL STRAIN

$$\varepsilon' = \frac{\Delta d_2}{d_2} = \frac{455 \times 10^{-6} \text{ in.}}{4.5 \text{ in.}} = 0.0001011$$

(a) INCREASE IN INNER DIAMETER

$$\Delta d_1 = \varepsilon' d_1 = (0.0001011)(3.9 \text{ in.})$$

$$= 394 \times 10^{-6} \text{ in.} \leftarrow$$

(b) INCREASE IN WALL THICKNESS

$$\Delta t = \varepsilon' t = (0.0001011)(0.3 \text{ in.})$$

$$= 30 \times 10^{-6} \text{ in.} \leftarrow$$

(c) POISSON'S RATIO

$$\text{Axial stress: } \sigma = \frac{P}{A}$$

$$A = \frac{\pi}{4}[d_2^2 - d_1^2] = \frac{\pi}{4}[(4.5 \text{ in.})^2 - (3.9 \text{ in.})^2]$$

$$= 3.9584 \text{ in.}^2$$

$$\sigma = \frac{P}{A} = \frac{40 \text{ k}}{3.9584 \text{ in.}^2}$$

$$= 10.105 \text{ ksi (compression)}$$

($\sigma < \sigma_y$; Hooke's law is valid)

Axial strain:

$$\varepsilon = \frac{\sigma}{E} = \frac{10,105 \text{ ksi}}{30,000 \text{ ksi}}$$

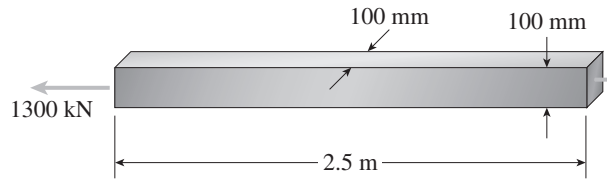
$$= 0.000337$$

$$\nu = \frac{\varepsilon'}{\varepsilon} = \frac{0.0001011}{0.000337}$$

$$= 0.30 \leftarrow$$

Problem 1.5-8 A steel bar of length 2.5 m with a square cross section 100 mm on each side is subjected to an axial tensile force of 1300 kN (see figure). Assume that $E = 200$ GPa and $\nu = 0.3$.

Determine the increase in volume of the bar.



Solution 1.5-8 Square bar in tension

Find increase in volume.

Length: $L = 2.5 \text{ m} = 2500 \text{ mm}$

Side: $b = 100 \text{ mm}$

Force: $P = 1300 \text{ kN}$

$E = 200 \text{ GPa}$ $\nu = 0.3$

AXIAL STRESS

$$\sigma = \frac{P}{A} = \frac{P}{b^2}$$

$$\sigma = \frac{1300 \text{ kN}}{(100 \text{ mm})^2} = 130 \text{ MPa}$$

Stress σ is less than the yield stress, so Hooke's law is valid.

AXIAL STRAIN

$$\begin{aligned} \varepsilon &= \frac{\sigma}{E} = \frac{130 \text{ MPa}}{200 \text{ GPa}} \\ &= 650 \times 10^{-6} \end{aligned}$$

INCREASE IN LENGTH

$$\begin{aligned} \Delta L &= \varepsilon L = (650 \times 10^{-6})(2500 \text{ mm}) \\ &= 1.625 \text{ mm} \end{aligned}$$

DECREASE IN SIDE DIMENSION

$$\begin{aligned} \varepsilon' &= \nu \varepsilon = 195 \times 10^{-6} \\ \Delta b &= \varepsilon' b = (195 \times 10^{-6})(100 \text{ mm}) \\ &= 0.0195 \text{ mm} \end{aligned}$$

FINAL DIMENSIONS

$$L_1 = L + \Delta L = 2501.625 \text{ mm}$$

$$b_1 = b - \Delta b = 99.9805 \text{ mm}$$

FINAL VOLUME

$$V_1 = L_1 b_1^2 = 25,006,490 \text{ mm}^3$$

INITIAL VOLUME

$$V = L b^2 = 25,000,000 \text{ mm}^3$$

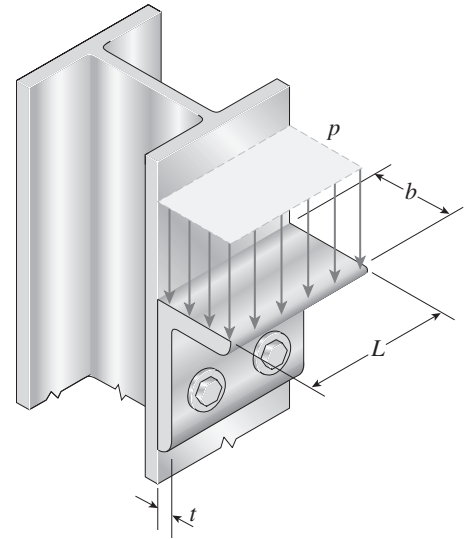
INCREASE IN VOLUME

$$\Delta V = V_1 - V = 6490 \text{ mm}^3 \quad \leftarrow$$

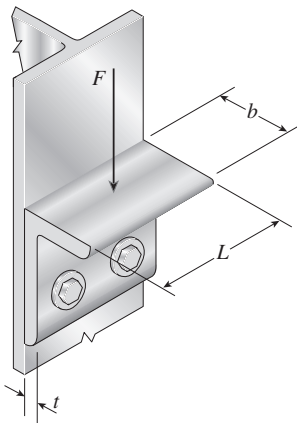
Shear Stress and Strain

Problem 1.6-1 An angle bracket having thickness $t = 0.5$ in. is attached to the flange of a column by two $\frac{5}{8}$ -inch diameter bolts (see figure). A uniformly distributed load acts on the top face of the bracket with a pressure $p = 300$ psi. The top face of the bracket has length $L = 6$ in. and width $b = 2.5$ in.

Determine the average bearing pressure σ_b between the angle bracket and the bolts and the average shear stress τ_{aver} in the bolts. (Disregard friction between the bracket and the column.)



Solution 1.6-1 Angle bracket bolted to a column



Two bolts

$$d = 0.625 \text{ in.}$$

$$t = \text{thickness of angle} = 0.5 \text{ in.}$$

$$b = 2.5 \text{ in.}$$

$$L = 6.0 \text{ in.}$$

$$\begin{aligned} p &= \text{pressure acting on top of the bracket} \\ &= 300 \text{ psi} \end{aligned}$$

$$\begin{aligned} F &= \text{resultant force acting on the bracket} \\ &= pbL = (300 \text{ psi})(2.5 \text{ in.})(6.0 \text{ in.}) = 4.50 \text{ k} \end{aligned}$$

BEARING PRESSURE BETWEEN BRACKET AND BOLTS

$$\begin{aligned} A_b &= \text{bearing area of one bolt} \\ &= dt = (0.625 \text{ in.})(0.5 \text{ in.}) = 0.3125 \text{ in.}^2 \end{aligned}$$

$$\sigma_b = \frac{F}{2A_b} = \frac{4.50 \text{ k}}{2(0.3125 \text{ in.}^2)} = 7.20 \text{ ksi} \leftarrow$$

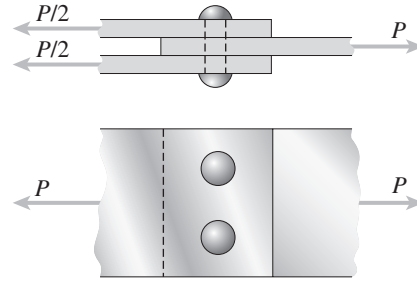
AVERAGE SHEAR STRESS IN THE BOLTS

$$\begin{aligned} A_s &= \text{Shear area of one bolt} \\ &= \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.625 \text{ in.})^2 = 0.3068 \text{ in.}^2 \end{aligned}$$

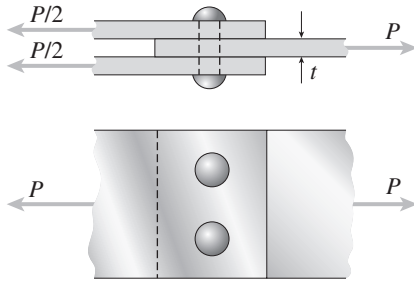
$$\tau_{aver} = \frac{F}{2A_s} = \frac{4.50 \text{ k}}{2(0.3068 \text{ in.}^2)} = 7.33 \text{ ksi} \leftarrow$$

Problem 1.6-2 Three steel plates, each 16 mm thick, are joined by two 20-mm diameter rivets as shown in the figure.

- (a) If the load $P = 50$ kN, what is the largest bearing stress acting on the rivets?
- (b) If the ultimate shear stress for the rivets is 180 MPa, what force P_{ult} is required to cause the rivets to fail in shear? (Disregard friction between the plates.)



Solution 1.6-2 Three plates joined by two rivets



$t =$ thickness of plates $= 16$ mm
 $d =$ diameter of rivets $= 20$ mm
 $P = 50$ kN

$\tau_{ULT} = 180$ MPa (for shear in the rivets)

(a) MAXIMUM BEARING STRESS ON THE RIVETS

Maximum stress occurs at the middle plate.

$$A_b = \text{bearing area for one rivet} \\ = dt$$

$$\sigma_b = \frac{P}{2A_b} = \frac{P}{2dt} = \frac{50 \text{ kN}}{2(20 \text{ mm})(16 \text{ mm})} \\ = 78.1 \text{ MPa} \leftarrow$$

(b) ULTIMATE LOAD IN SHEAR

$$\text{Shear force on two rivets} = \frac{P}{2}$$

$$\text{Shear force on one rivet} = \frac{P}{4}$$

Let $A =$ cross-sectional area of one rivet

$$\text{Shear stress } \tau = \frac{P/4}{A} = \frac{P}{4(\frac{\pi d^2}{4})} = \frac{P}{\pi d^2}$$

$$\text{or, } P = \pi d^2 \tau$$

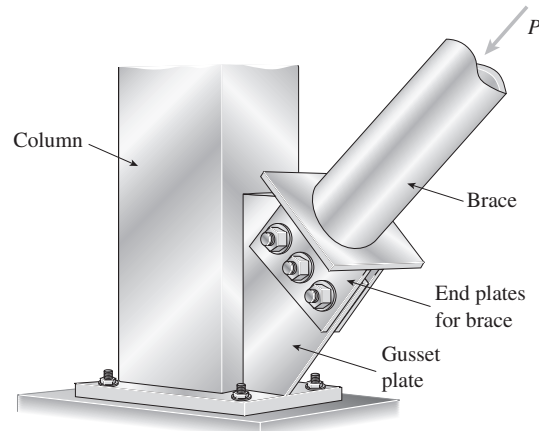
At the ultimate load:

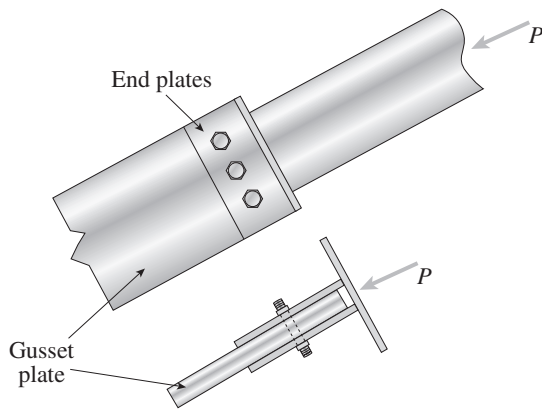
$$P_{ULT} = \pi d^2 \tau_{ULT} = \pi (20 \text{ mm})^2 (180 \text{ MPa}) \\ = 226 \text{ kN} \leftarrow$$

Problem 1.6-3 A bolted connection between a vertical column and a diagonal brace is shown in the figure. The connection consists of three $\frac{5}{8}$ -in. bolts that join two $\frac{1}{4}$ -in. end plates welded to the brace and a $\frac{5}{8}$ -in. gusset plate welded to the column. The compressive load P carried by the brace equals 8.0 k.

Determine the following quantities:

- (a) The average shear stress τ_{aver} in the bolts, and
- (b) The average bearing stress σ_b between the gusset plate and the bolts. (Disregard friction between the plates.)



Solution 1.6-3 Diagonal brace

3 bolts in double shear

P = compressive force in brace = 8.0 k

d = diameter of bolts = $\frac{5}{8}$ in. = 0.625 in.

t_1 = thickness of gusset plate

= $\frac{5}{8}$ in. = 0.625 in.

t_2 = thickness of end plates

= $\frac{1}{4}$ in. = 0.25 in.

(a) AVERAGE SHEAR STRESS IN THE BOLTS

A = cross-sectional area of one bolt

$$= \frac{\pi d^2}{4} = 0.3068 \text{ in.}^2$$

V = shear force acting on one bolt

$$= \frac{1}{3} \left(\frac{P}{2} \right) = \frac{P}{6}$$

$$\tau_{\text{aver}} = \frac{V}{A} = \frac{P}{6A} = \frac{8.0 \text{ k}}{6(0.3068 \text{ in.}^2)}$$

$$= 4350 \text{ psi} \leftarrow$$

(b) AVERAGE BEARING STRESS AGAINST GUSSET PLATE

A_b = bearing area of one bolt

$$= t_1 d = (0.625 \text{ in.})(0.625 \text{ in.}) = 0.3906 \text{ in.}^2$$

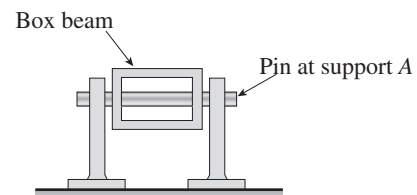
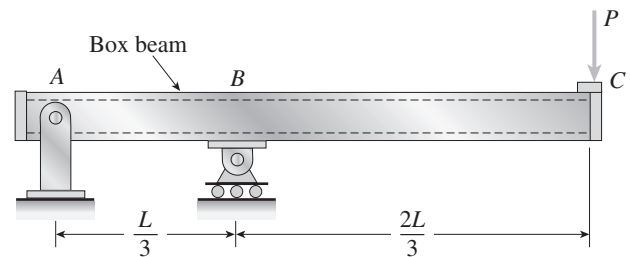
F = bearing force acting on gusset plate from one bolt

$$= \frac{P}{3}$$

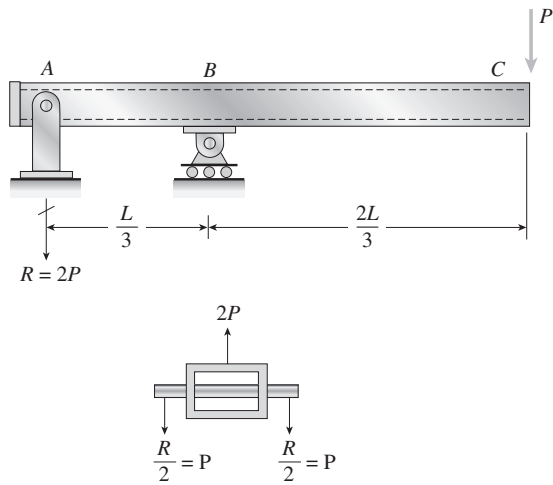
$$\sigma_b = \frac{F}{A_b} = \frac{P}{3A_b} = \frac{8.0 \text{ k}}{3(0.3906 \text{ in.}^2)} = 6830 \text{ psi} \leftarrow$$

Problem 1.6-4 A hollow box beam ABC of length L is supported at end A by a 20-mm diameter pin that passes through the beam and its supporting pedestals (see figure). The roller support at B is located at distance $L/3$ from end A .

- Determine the average shear stress in the pin due to a load P equal to 10 kN.
- Determine the average bearing stress between the pin and the box beam if the wall thickness of the beam is equal to 12 mm.



Solution 1.6-4 Hollow box beam



$P = 10 \text{ kN}$

$d = \text{diameter of pin} = 20 \text{ mm}$

$t = \text{Wall thickness of box beam} = 12 \text{ mm}$

(a) AVERAGE SHEAR STRESS IN PIN

Double shear

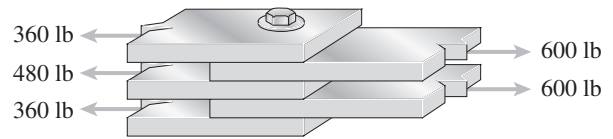
$$\tau_{\text{aver}} = \frac{2P}{2\left(\frac{\pi}{4}d^2\right)} = \frac{4P}{\pi d^2} = 31.8 \text{ MPa}$$

(b) AVERAGE BEARING STRESS ON PIN

$$\sigma_b = \frac{2P}{2(dt)} = \frac{P}{dt} = 41.7 \text{ MPa}$$

Problem 1.6-5 The connection shown in the figure consists of five steel plates, each $\frac{3}{16}$ in. thick, joined by a single $\frac{1}{4}$ -in. diameter bolt. The total load transferred between the plates is 1200 lb, distributed among the plates as shown.

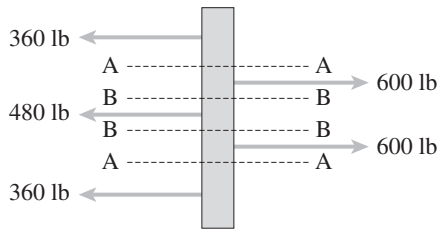
- (a) Calculate the largest shear stress in the bolt, disregarding friction between the plates.
- (b) Calculate the largest bearing stress acting against the bolt.



Solution 1.6-5 Plates joined by a bolt

$d = \text{diameter of bolt} = \frac{1}{4} \text{ in.}$

$t = \text{thickness of plates} = \frac{3}{16} \text{ in.}$



Free-body diagram of bolt

Section A – A: $V = 360 \text{ lb}$

Section B – B: $V = 240 \text{ lb}$

$V_{\text{max}} = \text{max. shear force in bolt}$
 $= 360 \text{ lb}$

(a) MAXIMUM SHEAR STRESS IN BOLT

$$\tau_{\text{max}} = \frac{V_{\text{max}}}{\frac{\pi d^2}{4}} = \frac{4V_{\text{max}}}{\pi d^2} = 7330 \text{ psi} \quad \leftarrow$$

(b) MAXIMUM BEARING STRESS

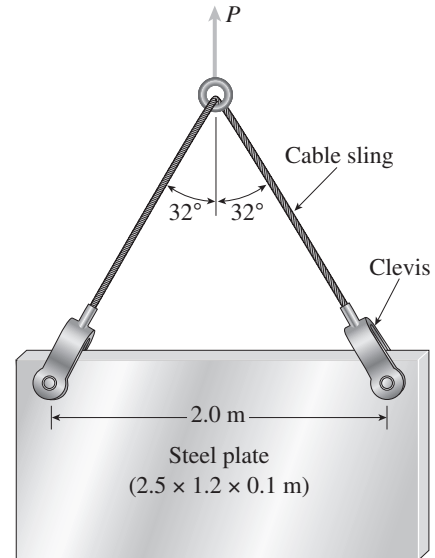
$F_{\text{max}} = \text{maximum force applied by a plate against the bolt}$

$F_{\text{max}} = 600 \text{ lb}$

$$\sigma_b = \frac{F_{\text{max}}}{dt} = 12,800 \text{ psi} \quad \leftarrow$$

Problem 1.6-6 A steel plate of dimensions $2.5 \times 1.2 \times 0.1$ m is hoisted by a cable sling that has a clevis at each end (see figure). The pins through the clevises are 18 mm in diameter and are located 2.0 m apart. Each half of the cable is at an angle of 32° to the vertical.

For these conditions, determine the average shear stress τ_{aver} in the pins and the average bearing stress σ_b between the steel plate and the pins.



Solution 1.6-6 Steel plate hoisted by a sling

Dimensions of plate: $2.5 \times 1.2 \times 0.1$ m

Volume of plate: $V = (2.5)(1.2)(0.1) \text{ m} = 0.300 \text{ m}^3$

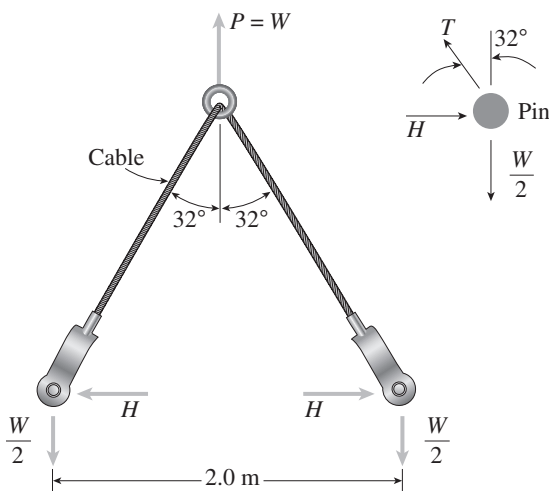
Weight density of steel: $\gamma = 77.0 \text{ kN/m}^3$

Weight of plate: $W = \gamma V = 23.10 \text{ kN}$

d = diameter of pin through clevis = 18 mm

t = thickness of plate = 0.1 m = 100 mm

FREE-BODY DIAGRAMS OF SLING AND PIN



TENSILE FORCE T IN CABLE

$$\sum F_{\text{vertical}} = 0 \quad \uparrow + \quad \downarrow -$$

$$T \cos 32^\circ - \frac{W}{2} = 0$$

$$T = \frac{W}{2 \cos 32^\circ} = \frac{23.10 \text{ kN}}{2 \cos 32^\circ} = 13.62 \text{ kN}$$

SHEAR STRESS IN THE PINS (DOUBLE SHEAR)

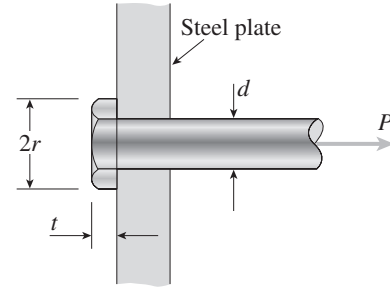
$$\begin{aligned} \tau_{\text{aver}} &= \frac{T}{2A_{\text{pin}}} = \frac{13.62 \text{ kN}}{2\left(\frac{\pi}{4}\right)(18 \text{ mm})^2} \\ &= 26.8 \text{ MPa} \end{aligned}$$

BEARING STRESS BETWEEN PLATE AND PINS

A_b = bearing area

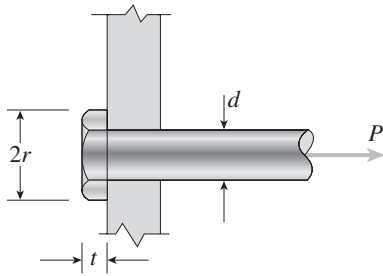
$$\begin{aligned} &= td \\ \sigma_b &= \frac{T}{td} = \frac{13.62 \text{ kN}}{(100 \text{ mm})(18 \text{ mm})} \\ &= 7.57 \text{ MPa} \end{aligned}$$

Problem 1.6-7 A special-purpose bolt of shank diameter $d = 0.50$ in. passes through a hole in a steel plate (see figure). The hexagonal head of the bolt bears directly against the steel plate. The radius of the circumscribed circle for the hexagon is $r = 0.40$ in. (which means that each side of the hexagon has length 0.40 in.). Also, the thickness t of the bolt head is 0.25 in. and the tensile force P in the bolt is 1000 lb.

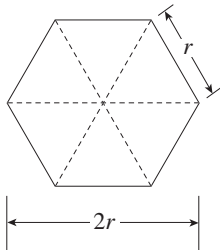


- (a) Determine the average bearing stress σ_b between the hexagonal head of the bolt and the plate.
- (b) Determine the average shear stress τ_{aver} in the head of the bolt.

Solution 1.6-7 Bolt in tension



$d = 0.50$ in.
 $r = 0.40$ in.
 $t = 0.25$ in.
 $P = 1000$ lb



Area of one equilateral triangle
 $= \frac{r^2 \sqrt{3}}{4}$

Area of hexagon
 $= \frac{3r^2 \sqrt{3}}{2}$

(a) BEARING STRESS BETWEEN BOLT HEAD AND PLATE

$A_b =$ bearing area

$A_b =$ area of hexagon minus area of bolt

$$= \frac{3r^2 \sqrt{3}}{2} - \frac{\pi d^2}{4}$$

$$A_b = \frac{3}{2}(0.40 \text{ in.})^2(\sqrt{3}) - \left(\frac{\pi}{4}\right)(0.50 \text{ in.})^2$$

$$= 0.4157 \text{ in.}^2 - 0.1963 \text{ in.}^2$$

$$= 0.2194 \text{ in.}^2$$

$$\sigma_b = \frac{P}{A_b} = \frac{1000 \text{ lb}}{0.2194 \text{ in.}^2} = 4560 \text{ psi} \leftarrow$$

(b) SHEAR STRESS IN HEAD OF BOLT

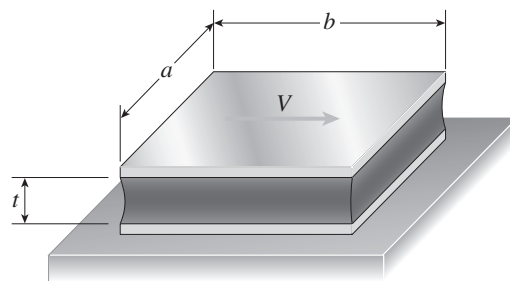
$A_s =$ shear area $A_s = \pi dt$

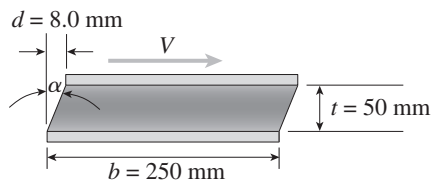
$$\tau_{\text{aver}} = \frac{P}{A_s} = \frac{P}{\pi dt} = \frac{1000 \text{ lb}}{\pi(0.50 \text{ in.})(0.25 \text{ in.})}$$

$$= 2550 \text{ psi} \leftarrow$$

Problem 1.6-8 An elastomeric bearing pad consisting of two steel plates bonded to a chloroprene elastomer (an artificial rubber) is subjected to a shear force V during a static loading test (see figure). The pad has dimensions $a = 150$ mm and $b = 250$ mm, and the elastomer has thickness $t = 50$ mm. When the force V equals 12 kN, the top plate is found to have displaced laterally 8.0 mm with respect to the bottom plate.

What is the shear modulus of elasticity G of the chloroprene?



Solution 1.6-8 Bearing pad subjected to shear

$$V = 12 \text{ kN}$$

$$\text{Width of pad: } a = 150 \text{ mm}$$

$$\text{Length of pad: } b = 250 \text{ mm}$$

$$d = 8.0 \text{ mm}$$

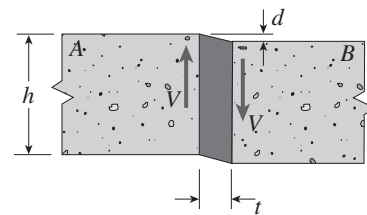
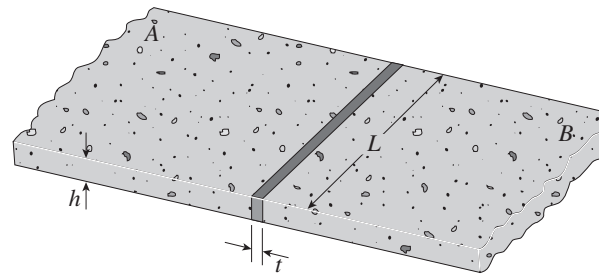
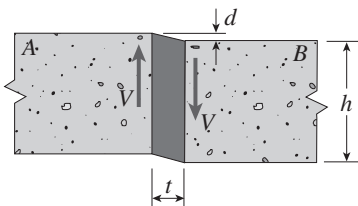
$$\tau_{\text{aver}} = \frac{V}{ab} = \frac{12 \text{ kN}}{(150 \text{ mm})(250 \text{ mm})} = 0.32 \text{ MPa}$$

$$\gamma_{\text{aver}} = \frac{d}{t} = \frac{8.0 \text{ mm}}{50 \text{ mm}} = 0.16$$

$$G = \frac{\tau}{\gamma} = \frac{0.32 \text{ MPa}}{0.16} = 2.0 \text{ MPa} \quad \leftarrow$$

Problem 1.6-9 A joint between two concrete slabs *A* and *B* is filled with a flexible epoxy that bonds securely to the concrete (see figure). The height of the joint is $h = 4.0$ in., its length is $L = 40$ in., and its thickness is $t = 0.5$ in. Under the action of shear forces V , the slabs displace vertically through the distance $d = 0.002$ in. relative to each other.

- What is the average shear strain γ_{aver} in the epoxy?
- What is the magnitude of the forces V if the shear modulus of elasticity G for the epoxy is 140 ksi?

**Solution 1.6-9 Epoxy Joint between concrete slabs**

$$h = 4.0 \text{ in.}$$

$$t = 0.5 \text{ in.}$$

$$L = 40 \text{ in.}$$

$$d = 0.002 \text{ in.}$$

$$G = 140 \text{ ksi}$$

- AVERAGE SHEAR STRAIN

$$\gamma_{\text{aver}} = \frac{d}{t} = 0.004 \quad \leftarrow$$

- SHEAR FORCES V

$$\text{Average shear stress: } \tau_{\text{aver}} = G\gamma_{\text{aver}}$$

$$\begin{aligned} V &= \tau_{\text{aver}}(hL) = G\gamma_{\text{aver}}(hL) \\ &= (140 \text{ ksi})(0.004)(4.0 \text{ in.})(40 \text{ in.}) \\ &= 89.6 \text{ k} \quad \leftarrow \end{aligned}$$